29 Inequalities

After studying this lesson, you can get a good understanding of

- ★ solving inequalities of the form $x \pm a \leq b$
- ★ solving inequalities of the form $x \pm a \geq b$
- ★ solving inequalities of the form $ax \leq b$, $a \neq 0$
- ★ solving inequalities of the form $ax \geq b$, $a \neq 0$
- ★ representing solutions on a number line.

Activity 29.1

Do the following exercise to revise what you have learnt about inequalities

(1) Fill in the blanks using the symbols $>$, $<$ or $=$

(i) 3 ----- 5 (ii) (– 2) ----- 4 (iii) 4 ----- (– 5)
(iv) (– 2) ---- (– 3) (v) (– 5) ----- 5 (vi) 8 ---- 8

(2) Write the given expressions as inequalities.

(i) x is a number greater than or equal to five

(ii) b is less than 3

(iii) y is a positive number

(iv) x is a negative number

(3) Write the given inequalities in words.

(i) $a \geq 5$ (ii) $x \leq -4$ (iii) $y > 3$ (iv) $x \leq -6$ (v) $x > 0$

(4) Solve the fllowing inequalities and show the solutions on a number line. Here x is a positive integer.

(i) $x + 5 < 6$ (ii) $x - 5 > 3$ (iii) $x - 4 < 3$
(iv) $x + 5 > 7$ (v) $2x > 8$ (vi) $3x < 12$

29.1 Solution of algebraic inequalities

Example 1

Solve the inequality $x + 7 \leq 6$ and show the solution on a number line.

$x + 7 \leq 6$

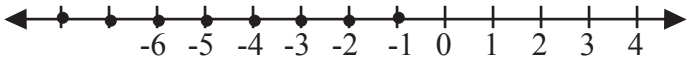
$x + 7 - 7 \leq 6 - 7$ (By adding -7 to both sides of the inequality)

$x \leq -1$

$\therefore x \leq -1$ The integer solution is

$x = -1, -2, -3,$ ----------

Representing the solution on a number line.



When a number is added to both sides or subtracted from both sides of an inequality, the inequality does not change.

$4 > 2$

$4 + 2 > 2 + 2$ (Adding + 2 to both sides)

$6 > 4$

The inequality has not changed

$-4 < -2$

$-4 - 3 < -2 - 3$ (Adding -3 to both sides)

$-7 < -5$

The inequality has not changed

Exercise 29.1

(1) Solve the following inequalities

(i) $x + 6 \leq 5$ (ii) $x - 7 \geq -3$ (iii) $x + 8 \leq -6$

(iv) $x - 5 \geq 10$ (v) $x - 10 \leq -15$ (vi) $x + 16 \leq 10$

(2) Solve the following inequalities and show the integer solutions of each inequality on a number line.

(i) $x + 5 \leq -4$ (ii) $x - 12 \geq 7$ (iii) $2 + x \geq -4$

(iv) $x - 1 \leq -2$ (v) $x + 5 \geq 6$ (vi) $x - 6 \geq -6$

29.2 Solving inequalities of the form $ax \geq b$, $ax \leq b$

When an inequality is multiplied or divided by a positive number, the sign of the inequality will not change.

$4 > 2$

$\frac{4}{2} > \frac{2}{2}$ (Dividing both sides by 2)

$2 > 1$ Inequality did not change

$-4 > -8$

$2 \times (-4) > (-8) \times 2$ (Multiplying both sides by 2)

$-8 > -16$ Inequality did not change

Let us find what happens when an inequality is multiplied by a negative number.

$4 > 2$

$(-2) \times 4 > (-2) \times 2$ (Multiplying both sides by (-2))

When multiplied by (-2) we get

$-8 > -4$ which is not true

It should be changed as $-8 < -4$

Similarly, we will divide another inequality by a negative number.

$$-6 > -8$$

$$\frac{-6}{-2} > \frac{-8}{-2}$$ (Dividing both sides by (−2))

When divided by (−2) we get

$+3 > +4$ which is not true

It should be changed as $3 < 4$

When an inequality is divided or multiplied by a negative number, the inequality changes.

When solving inequalities, attention should be made to the following facts.

★ **When a positive or a negative number is added to both sides of an inequality, the inequality remains unchanged.**

★ **When both sides of an inequality is multiplied or divided by a positive number, the inequality remains unchanged.**

★ **When both sides of an inequality is multiplied or divided by a negative number, the inequality sign changes.**

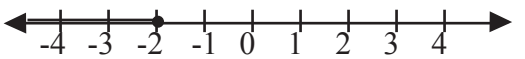
Example 2

Solve the inequality $-5x \geq 10$ and represent the solution on a number line.

$$-5x \geq 10$$

$$\frac{-5}{-5} \leq \frac{10}{-5}$$ (Inequality sign changes when divided by -5)

$$x \leq -2$$



Exercise 29.2

(1) Find the integer solution of each inequality given below.

(i) $2x \leq 8$ (ii) $-3x \geq 12$ (iii) $-4x > 0$

(iv) $-5x \geq -8$ (v) $-6x \leq -15$ (vi) $-4x \geq 18$

(2) Find the interger solution of each inequality and represent the solution on a number line.

(i) $x + 4 \geq 5$ (ii) $x + 5 \geq 2$ (iii) $3x - 4 > 10$

(vi) $5x - 3 < -18$ (v) $5 - 4x \leq 3$ (vi) $7 - x \geq 15$

(3) Find the solutions of the following inequalities and represent them on number lines.

(i) $x + 4 \geq 5$ (ii) $-3x + 6 \leq -3$ (iii) $-2x - 4 \leq -6$

(iv) $\frac{-x}{3} + 1 \geq -2$ (v) $\frac{-2x}{3} + 4 \geq 0$ (vi) $3 - 4x \geq -5$

(4) Write the minimum whole number which satisfies the inequality.

$2x + 3 \leq x + 5$

(5) Write the solution set of the inequality $5x + 4x \leq 6x + 8$. Write the highest whole number which satisfies the solution.

(6) Write the solution set of the inequality $7x - 5 \leq 3x + 15$. Write the highest whole number and the lowest whole number which satisfies the solution.

Summary

★ The solution of an algebraic inequality can be illustrated on a number line.

★ When any number is added to both sides or subtracted from both sides of an inequality of the form $x \pm a \leq$ b or $x \pm a \geq$ b the inequality remains unchanged.

★ Wen both sides of inequalities of the form $ax \geq$ b or $an \leq$ b are multiplied or divided by a positive number the inequalities remain unchanged.

★ When both sides of inequalities of the form ax ≥ b or ax ≤ b are multiplied or divided by a negative number the signs of the inequalities change.