1 Number Patterns

After studying this lesson, you can get a good understanding of

- ★ number sequences and the general term of a number sequence.
- ★ number sequences constructed by odd numbers, even numbers, square numbers, triangular numbers and their multiples.
- ★ finding the number sequence by using the general term.

1.1 Multiplication through Patterns

You may remember that the set of natural numbers can be divided into two groups such as odd numbers and even numbers, and the numbers which are divisible by two are called even numbers and the numbers which are not divisible by two are called odd numbers. A natural number can be obtained by dividing a suitable even number by two. Multiplication of any natural number by two always gives an even number. All even numbers are multiples of two. Let n be a natural number. Then 2n is an even number.

When $n = 1$ $\quad 2n = 2 \times 1 = 2$
When $n = 2$ $\quad 2n = 2 \times 2 = 4$
When $n = 3$ $\quad 2n = 2 \times 3 = 6$
When $n = 4$ $\quad 2n = 2 \times 4 = 8$

Now let us consider the even numbers, 2 , 4 , 6 , 8. All these numbers are multiples of two. Examine the way how they are ordered in a certain pattern. 1^{st} term of the number pattern when $(n = 1)$ is 2, 2^{nd} term when $(n = 2)$ is 4, 3^{rd} term when $(n = 3)$ is 6, 4^{th} term when $(n = 4)$ is 8.

Accordingly the n^{th} term of this sequence is 2n. This is called the **general term** of the above sequence.

Now consider the number sequence constructed by multiples of 3.
3 , 6 , 9 , 12 , 15 , ---

The n^{th} term or the general term of this sequence is 3n.

Also, the n^{th} term or the general term of the number sequence 4 , 8 , 12 , 16 , --- constructed by multiples of 4 is 4n.

Accordingly, the general term representing multiples of 2 starting from 2 (even numbers) $\rightarrow 2n$

The general term representing multiples of 3 starting from 3 $\rightarrow$ 3n
The general term representing multiples of 4 starting from 4 $\rightarrow$ 4n

So, we can conclude that, the general term of sequences constructed by multiples of various numbers can be represented by a symbol.

1, 3, 5, 7, 9, --- are odd numbers. They are not multiples of a certain number. But in that sequence the difference between any two consecutive numbers is 2. In such sequences, the general term can be found in terms of multiples of 2. Study the following analysis.

1^{st} term $= 1 = 2\times1 - 1$
2^{nd} term $= 3 = 2\times2 - 1$
3^{rd} term $= 5 = 2\times3 - 1$
4^{th} term $= 7 = 2\times4 - 1$
10^{th} term $= ? = 2\times10 - 1 = 19$
15^{th} term $= ? = 2\times15 - 1 = 29$
18^{th} term $= ? = 2\times18 - 1 = 37$
n^{th} term $= ? = 2\times n - 1 = 2n - 1$

Accordingly, the general term or the n^{th} term of odd numbers is 2n - 1. The relevant odd number can be obtained by substituting the relevant number for n to the general term 2n - 1.

Number sequence	**The name**	**General term**
2, 4, 6, 8, 10, ---	even numbers	2n
3, 6, 9, 12, 15, ---	multiples of 3	3n
4, 8, 12, 16, 20, ---	multiples of 4	4n
1, 3, 5, 7, 9, 11, ---	odd numbers	2n – 1

Example 1

What is the 18^{th} even number?

The general term of even numbers is 2n.
When $n = 18$, $2n = 2\times18 = \underline{\underline{36}}$
$\therefore$ 18^{th} even number is 36

Example 2

In the sequence 2, 4, 6, 8, --- which term is 156 ?
This sequence is the sequence of even numbers. The general term of it is 2n. If 156 is the n^{th} term then,

$$2n = 156$$

$$n = \frac{156}{2} = \underline{\underline{78}}$$

$\therefore$ In the above sequence 156 is the 78^{th} term.

Example 3

What is the 27^{th} odd number?

The general term of odd numbers is $2n - 1$

When $n = 27$, $2n - 1 = 2 \times 27 - 1$

$= 54 - 1$

$= 53$

$\therefore$ The 27^{th} odd number is $= \underline{\underline{53}}$

Example 4

Find the 36^{th} term of the number sequence 1, 3, 5, 7, -----

The general term of the number sequence is $2n - 1$

when $n = 36$, $2n - 1 = 2 \times 36 - 1$

$= 72 - 1$

$= \underline{\underline{71}}$

Example 5

Which odd number term is 99?

If 99 is the n^{th} odd number then

$2n - 1 = 99$

$2n = 100$

$n = \underline{\underline{50}}$

$\therefore$ The 50^{th} odd number is $\underline{\underline{99}}$

Example 6

Find the 45^{th} multiple of 4

The general term representing a multiple of 4 is 4n

when $n = 45$, $4n = 4 \times 45$

$= 180$

$\therefore$ The 45^{th} multiple of 4

$= \underline{\underline{180}}$

Example 7

Find the 17^{th} term of number sequence 6, 12, 18, 24, ----

This number sequence represents multiples of 6. The general term representing multiples of 6 is 6n

when, $n = 17$,

$6n = 6 \times 17 = 102$ $\therefore$ 17^{th} term $= \underline{\underline{102}}$

Exercise 1.1

(1) What is the 29th even number?

(2) Find the 13th multiple of 5

(3) Which even number is 84?

(4) Which even number is 176?

(5) What is the 48th multiple of 3?

(6) What is the even number larger than 100 but closest to 100?

(7) What is the 24th odd number?

(8) Which odd number is 195?

(9) What is the largest odd number less than 300 which odd number is that?

(10) (i) Which even number is 204?

(ii) Which multiple of 3 is 204?

(iii) Which multiple of 4 is 204?

(iv) Which multiple of 6 is 204?

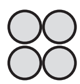
(v) Which multiple of 12 is 204?

1.2 Square numbers and Triangular numbers

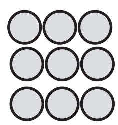
Square numbers

$1 \times 1 = 1^2 = 1$

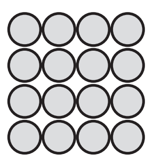
$2 \times 2 = 2^2 = 4$



$3 \times 3 = 3^3 = 9$



$4 \times 4 = 4^4 = 16$

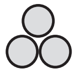


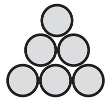
When a natural number is multiplied by itself, the number obtained as the answer is a square number. Square number pattern can be formed by squaring numbers. If any number is represented by n, the relevant square number can be represented as n^2 accordingly.

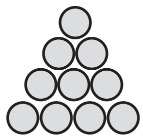
In the sequence containing square numbers 1, 4, 9, 16, 25, ----- the general term is n^2.

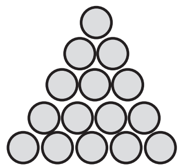
Triangular numbers

$\frac{1\times2}{2}=1 \rightarrow$  $\rightarrow 1$

$\frac{2\times3}{2}=3 \rightarrow$  $\rightarrow 1+2$

$\frac{3\times4}{2}=6 \rightarrow$  $\rightarrow 1+2+3$

$\frac{4\times5}{2}=10 \rightarrow$  $\rightarrow 1+2+3+4$

$\frac{5\times6}{2}=15 \rightarrow$  $\rightarrow 1+2+3+4+5$

The value of half of the product of two consecutive natural numbers is a triangular number.

Triangular patterns can be prepared by those values. Now if any natural number is represented by n the consecutive natural number can be represented as $n+1$. Hence the general term of a sequence of triangular numbers can be written as $\frac{n(n+1)}{2}$.

By examining the right hand side of the above pattern, it can be seen that the value of a triangular number represents the sum of groups of natural numbers starting from 1.

See the table below

The n^{th} triangular number	The sum of the natural numbers from 1 to n	$\frac{n(n+1)}{2}$
5^{th} triangular number	The sum of the natural numbers from 1 to 5	$\frac{5\times6}{2}=15$
10^{th} triangular number	The sum of the natural numbers from 1 to 10	$\frac{10\times11}{2}=55$
60^{th} triangular number	Sum of the natural numbers from 1 to 60	$\frac{60\times61}{2}=1830$

Exercise 1.2

(1) Write all the square numbers from 1 to 625

(2) Which square number is 225?

(3) If the first square number is considered as S_1, second as S_2 and the third is S_3 and so on, find the values of

(i) S_8 (ii) S_{20} (iii) S_{16}

(4) Select and write the correct ones out of the following

(i) $S_3 + S_4 = S_5$ (ii) $S_4 + S_5 = S_6$ (iii) $S_5 + S_{12} = S_{13}$

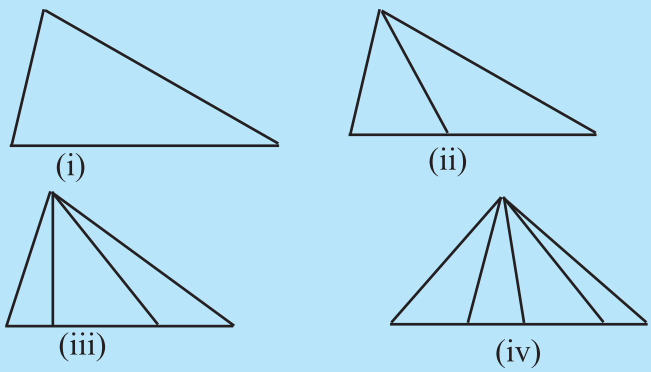
(iv) $S_6 + S_8 = S_{10}$ (v) $S_{25} + S_{24} = S_7$

(5) By considering the first triangular numbers as T_1, the second as T_2, the third as T_3 and so on find the values of

(i) T_{12} (ii) T_{20} (iii) T_{36}

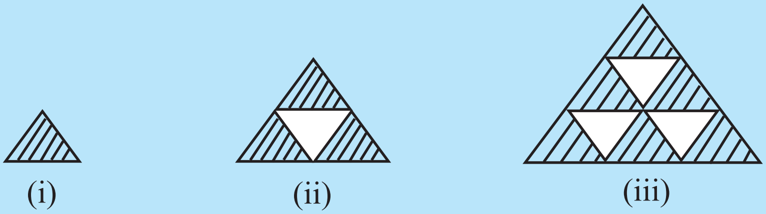
(6) Find the sum of the natural numbers from 1 to 25 without adding those numbers in order. Which triangular number is that?

(7)



Write the number of triangles in each of the above figures. If seven such figures are considered, how many triangles would be there in the (vii)[th] figure?

(8) Write the number of shaded triangles in each of these figuers. How many shaded triangles should be there in the (xii)[th] figure?



1.3 Other number Patterns in terms of multiples

8, 10, 12, 14, 16, -----

In this sequence all the numbers are even numbers. But this does not include the even numbers 2, 4 and 6. Let us inquire as to how the general term could be found for a sequence of this nature. Since the difference between two consecutive terms in this is 2 the general term could be found in terms of 2.

Term of the sequence	Multiples of 2	Analysis of the terms of the sequence in terms of multiples of 2
8	2	$2 \times 1+6 = 2 + 6$
10	4	$2 \times 2+6 = 4 + 6$
12	6	$2 \times 3+6 = 6 + 6$
14	8	$2 \times 4+6 = 8 + 6$
16	10	$2 \times 5+6 = 10+ 6$
The general term $\rightarrow 2 \times n + 6 = 2n + 6$		

Exercise 1.3

(1) Find the general term of each of the following sequences.

(i) 6, 9, 12, 15, 18, ----

(ii) 30, 35, 40, 45, -----

(iii) 0, 2, 4, 6, 8, 10, -----

(iv) 20, 24, 28, 32, -----

(v) 12 , 14 , 16 , 18 , 20 ,-----

(2) The general terms of a few number sequences are given below. Write the first five terms of each sequence

(i) $10n$ (ii) $2n + 18$ (iii) $4n - 4$ (iv) $2n + 1$ (v) $22 - 4n$

summary

- ★ The general term of any multiple can be given using n. The general term of the multiples of 2 is 2n, the general term of multiples of 3 is 3n, the general term of multiples of 4 is 4 n and so on.

- ★ The general term of a number sequence of which the difference between two consecutive numbers is a constant, can be obtained in terms of the multiples revelant to that difference between the two consecutive numbers.

- ★ The general term of the square number sequence is n^2.
- ★ The general term of the triangular number sequence is $\frac{n(n+1)}{2}$