MATHEMATICS

Grade 9

Part - I

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The National Anthem of Sri Lanka

Sri Lanka Matha
Apa Sri Lanka Namo Namo Namo Namo Matha
Sundara siri barinee, surendi athi sobamana Lanka
Dhanya dhanaya neka mal palaturu piri jaya bhoomiya ramya
Apa hata sepa siri setha sadana jeewanaye matha
Piliganu mena apa bhakthi pooja Namo Namo Matha
Apa Sri Lanka Namo Namo Namo Namo Matha
Oba we apa vidya
Obamaya apa sathya
Oba we apa shakthi
Apa hada thula bhakthi
Oba apa aloke
Apage anuprane
Oba apa jeevana we
Apa mukthiya oba we
Nava jeevana demine, nithina apa pubudukaran matha
Gnana veerya vadawamina regena yanu mana jaya bhoomi kara
Eka mavakage daru kela bevina
Yamu yamu vee nopama
Prema vada sema bheda durerada
Namo, Namo Matha
Apa Sri Lanka Namo Namo Namo Namo Matha

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ආනන්ද සමරකෝන්

ஒரு தாய் மக்கள் நாமாவோம்
ஒன்றே நாம் வாழும் இல்லம்
நன்றே உடலில் ஓடும்
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ஒற்றுமை சிறக்க வாழ்ந்திடுதல்
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யான்று மழியாச் செல்வமன்றோ.

ஆனந்த சமரக்கோன்
கவிதையின் பெயர்ப்பு.

Being innovative, changing with right knowledge
Be a light to the country as well as to the world.

Message from the Hon. Minister of Education

The past two decades have been significant in the world history due to changes that took place in technology. The present students face a lot of new challenges along with the rapid development of Information Technology, communication and other related fields. The manner of career opportunities are liable to change specifically in the near future. In such an environment, with a new technological and intellectual society, thousands of innovative career opportunities would be created. To win those challenges, it is the responsibility of the Sri Lankan Government and myself, as the Minister of Education, to empower you all.

This book is a product of free education. Your aim must be to use this book properly and acquire the necessary knowledge out of it. The government in turn is able to provide free textbooks to you, as a result of the commitment and labour of your parents and elders.

Since we have understood that the education is crucial in deciding the future of a country, the government has taken steps to change curriculum to suit the rapid changes of the technological world. Hence, you have to dedicate yourselves to become productive citizens. I believe that the knowledge this book provides will suffice your aim.

It is your duty to give a proper value to the money spent by the government on your education. Also you should understand that education determines your future. Make sure that you reach the optimum social stratum through education.

I congratulate you to enjoy the benefits of free education and bloom as an honoured citizen who takes the name of Sri Lanka to the world.

Akila Viraj Kariyawasam
Minister of Education

Foreword

The educational objectives of the contemporary world are becoming more complex along with the economic, social, cultural and technological development. The learning and teaching process too is changing in relation to human experiences, technological differences, research and new indices. Therefore, it is required to produce the textbook by including subject related information according to the objectives in the syllabus in order to maintain the teaching process by organizing learning experiences that suit to the learner needs. The textbook is not merely a learning tool for the learner. It is a blessing that contributes to obtain a higher education along with a development of conduct and attitudes, to develop values and to obtain learning experiences.

The government in its realization of the concept of free education has offered you all the textbooks from grades 1-11. I would like to remind you that you should make the maximum use of these textbooks and protect them well. I sincerely hope that this textbook would assist you to obtain the expertise to become a virtuous citizen with a complete personality who would be a valuable asset to the country.

I would like to bestow my sincere thanks on the members of the editorial and writer boards as well as on the staff of the Educational Publications Department who have strived to offer this textbook to you.

W. M. Jayantha Wickramanayaka,
Commissioner General of Educational Publications,
Educational Publications Department,
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2019.04.10

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Message of the Board of Compilers

This textbook has been compiled in accordance with the new syllabus to be implemented from 2015.

Textbooks are compiled for students. Therefore, we have made an attempt to compile this textbook in a simple and detailed manner making it possible for you to read and understand it on your own.

We have included descriptions, activities and examples to introduce the subject concepts in an attractive manner and to establish them. Moreover, activities are organized from simple to complex to develop an interest to do them.

We have used the terms related to mathematical concepts in accordance with the glossary of technical terms of mathematics compiled by the Department of Official Languages.

Some subject matter learnt during the earlier grades is necessary to learn the subject content in the grade 9 syllabus. Thus, review exercises are included at the beginning of each chapter to revise previous knowledge. You will be prepared by them for the subject content of grade 9.

You will gain maximum benefit from this textbook by reading the chapters and doing the review exercises of each chapter even before your teacher teaches them in the classroom.

We hope that studying mathematics will be an interesting, joyful and productive experience.

Board of Compilers

Contents

1 Number Patterns

By studying this lesson you will be able to;

- develop the general term of a number pattern with the same difference between adjacent terms,
- develop the number pattern when the general term is known,
- solve problems associated with number patterns.

Introduction to number patterns

Several number patterns are given below.

i. 3, 3, 3, 3, 3, ...

ii. 2, 4, 6, 8, 10, ...

iii. 5, 8, 11, 14, 17, ...

iv. 2, 4, 8, 16, 32, ...

v. 1, 1, 2, 2, 3, 3, 4, 4, 5, 5, ...

vi. 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, ...

The first number pattern is very simple. Every number of this number pattern is 3. While the first number of the second number pattern is 2, all the numbers thereafter are obtained by adding 2 to the previous number.

While the first number of the third number pattern is 5, all the numbers thereafter are obtained by adding 3 to the previous number.

While the first number of the fourth number pattern is 2, all the numbers thereafter are obtained by multiplying the previous number by 2.

The fifth and sixth number patterns have characteristics which are inherent to them.

The numbers of a number pattern are called "terms".

For example, each term of the first number pattern is 3.

The first term of the second number pattern is 2, the second term is 4, the third term is 6, etc. In this pattern, each term which comes after the first term is obtained by adding 2 to the previous term.

The first term of the third number pattern is 5, the second term is 8, the third term is 11, etc. In this pattern, each term which comes after the first term is obtained by adding 3 to the previous term.

In the fourth number pattern, each term which comes after the first term is obtained by multiplying the previous term by 2.

The ways in which the terms of the fifth and sixth number patterns are obtained can also be described as above. However, the descriptions will be more complicated.

Observe that the terms of the number patterns given above are separated by commas and that there are three dots (ellipsis) at the end of each number pattern. This is how number patterns are usually written. The three dots indicate that the number pattern continues.

In mathematics, the word "sequence" is used for the word "pattern".
Accordingly, six "number sequences" (or simply "sequences") are given above.
The order of the terms of a sequence is important.

For example, although the sequence 1, 1, 2, 2, 3, 3, 4, 4, 5, 5, 6, 6, ...
and the sequence 1, 2, 1, 2, 3, 4, 3, 4, 5, 6, 5, 6, ... consist of the same numbers, they are two different sequences.

In the above examples of sequences, only a few initial terms are given. However, it is incorrect to presume the pattern of the sequence by considering only a few initial terms.
For example,

1, 2, 3, 4, 5, 1, 2, 3, 4, 5, 1, 2, 3, 4, 5, ...

is a number pattern; that is, a sequence. If only the first five terms of the sequence are given (that is, 1, 2, 3, 4, 5, …), and a person is asked what the next term is, one may be provided with the incorrect answer 6. Hence, asking for the next term (or next few terms) after giving only the first few terms of a sequence is mathematically incorrect.

A method of describing a sequence accurately is by providing a rule by which each term of the sequence can be calculated.

The uniqueness (or characteristic) of the second and third sequences of the six sequences given above can be explained as follows.

In the second sequence, every term which comes after the first term is obtained by adding the constant value 2 to the previous term. This can be illustrated as follows.

$$\underbrace{2 \quad 4}_{+2}\!\!\underbrace{\quad 6}_{+2}\!\!\underbrace{\quad 8}_{+2}\!\!\underbrace{\quad 10}_{+2}$$

In the third sequence, every term which comes after the first term is obtained by adding the constant value 3 to the previous term. This can be illustrated as follows.

$$\underbrace{5 \quad 8}_{+3}\!\!\underbrace{\quad 11}_{+3}\!\!\underbrace{\quad 14}_{+3}\!\!\underbrace{\quad 17}_{+3}$$

Here, the meaning of "constant value" is "the value remains unchanged".

The characteristic which is common to both these patterns can be described as follows.

"The value obtained by subtracting the previous term from any term (except the first term) is a constant (that is, a constant value)."

The value of this constant is 2 for the sequence 2, 4, 6, 8, 10, …
(since $4 - 2 = 6 - 4 = 8 - 6 = 10 - 8 = 2$).

The value of this constant is 3 for the sequence 5, 8, 11, 14, 17, …
(since $8 - 5 = 11 - 8 = 14 - 11 = 17 - 14 = 3$).

Let us study further about sequences of which the difference between every pair of consecutive terms is a constant value.

This constant value is known as the common difference of the sequence. Accordingly,

common difference = any term except the first term – the previous term

It can be seen that the first sequence 3, 3, 3, 3, 3, … also has the same characteristic.

$$\begin{array}{ccccccccc} 3 & & 3 & & 3 & & 3 & & 3 \\ & +0 & & +0 & & +0 & & +0 & \end{array}$$

Here, the constant value added (that is the common difference) is 0.

Another sequence with the same characteristic is given below.

$$\begin{array}{ccccccccc} 17 & & 12 & & 7 & & 2 & & -3\ldots \\ & -5 & & -5 & & -5 & & -5 & \end{array}$$

The first term of this sequence is 17. Every term which comes thereafter is obtained by subtracting 5 from the previous term. That is, by adding −5 to the previous term. Accordingly, the common difference of this sequence is −5.

Common difference $= 12 - 17 = 7 - 12 = 2 - 7 = -3 - 2 = -5$.

If the value of the common difference and the first term of a sequence with a common difference are known, the first few terms of the sequence can be written easily. A couple of examples of such sequences are given below.

Example 1

The first three terms of the sequence with first term 4 and common difference 3 are 4, 7 and 10.

Example 2

The first five terms of the sequence with first term 7 and common difference − 4 are 7, 3. −1, −5 and −9.

The first few terms of a sequence with a common difference can be written easily, when the first term and common difference are given. But it is not so easy to find, say the 50th term or the 834th term. The reason is because 50 and 834 are fairly large numbers.
It is important to know the general term of a sequence to be able to find any term of the sequence easily. Now let us see what is meant by the general term.

The general term of a number pattern

First, let us introduce a specific notation to denote the terms of a sequence. For a given sequence, let us denote

the first term by T_1,
the second term by T_2,
the third term by T_3, etc.

For example, with regard to the sequence

$$5, 11, 17, 23, \ldots$$

we can indicate the terms as follows:

the first term, $T_1 = 5$
the second term, $T_2 = 11$
the third term, $T_3 = 17$
the fourth term, $T_4 = 23$

It is very important to consider the nth term of a sequence, as is usually done in mathematics. Here, n represents any positive integer. The reason for this is that the values n can assume are positive integers such as 1, 2, 3, ... The $\frac{1}{2}$th term, the −4th term, the 3.5th term, etc., have no meaning. When considering a value n, the corresponding nth term is denoted by T_n. This is called the **general term** of the sequence. Accordingly,

the general term (nth term) of a sequence is denoted by T_n.

1.1 Developing the number sequence when the general term is given

In the previous section we learnt the notation that is used to denote the terms of a sequence, in particular, the general term. Now, through a couple of examples, let us consider how to develop the number sequence and how to find a named term of the sequence, when the general term is given.

Example 1

Consider the number sequence with general term $T_n = 2n + 3$.

(i) Write the first three terms of this sequence.
(ii) Find the 20th term.
(iii) Which term is equal to 123?
(iv) Find the $(n + 1)$th term in terms of n.

(i) Since the general term $T_n = 2n + 3$,
when $n = 1$; the first term $T_1 = (2 \times 1) + 3 = 2 + 3 = 5$,
when $n = 2$; the second term $T_2 = (2 \times 2) + 3 = 4 + 3 = 7$,
when $n = 3$; the third term $T_3 = (2 \times 3) + 3 = 6 + 3 = 9$.

Therefore, the first three terms of this number pattern are 5, 7, 9.

(ii) The 20th term is obtained by substituting $n = 20$ in $2n + 3$.

The 20th term, $T_{20} = (2 \times 20) + 3 = 40 + 3$
$= 43$

Therefore, the 20th term is 43.

(iii) Let us assume that the nth term is 123.

$$\begin{aligned} \text{Then, } 2n + 3 &= 123 \\ 2n + 3 - 3 &= 123 - 3 \\ 2n &= 120 \\ n &= \frac{120}{2} \\ &= 60 \end{aligned}$$

Therefore, 123 is the 60th term of the number pattern.

(iv) In order to obtain the $(n + 1)$th term, let us substitute $(n + 1)$ for n.
The $(n + 1)$th term,

$$\begin{aligned} T_{n+1} &= 2\,(n + 1) + 3 \\ &= 2n + 2 + 3 \\ &= 2n + 5 \end{aligned}$$

Therefore, the $(n + 1)$th term, in terms of n, is $2n + 5$.

Example 2

Consider the number pattern with general term $T_n = 56 - 4n$.

(i) Write the first three terms of this number pattern.
(ii) Find the 12th term.
(iii) Show that 0 is a term of this number pattern.
(iv) Show that 18 is not a term of this number pattern.

(i) Since the general term $T_n = 56 - 4n$,

when $n = 1$; the first term $T_1 = 56 - (4 \times 1) = 56 - 4 = 52$
when $n = 2$; the second term $T_2 = 56 - (4 \times 2) = 56 - 8 = 48$
when $n = 3$; the third term $T_3 = 56 - (4 \times 3) = 56 - 12 = 44$
Therefore, the first three terms of the number pattern are 52, 48, 44.

(ii)The 12th term $= 56 - 4 \times 12$
$= 56 - 48$
$= 8$

(iii) If 0 is a term of the number pattern, then for some integer n, we have

$$56 - 4n = 0.$$
$\therefore\ 56 - 4n + 4n = 4n$ (adding $4n$ to both sides)
$$\frac{56}{4} = \frac{4n}{4}$$
$$14 = n$$
$$n = 14$$
The 14th term of the number pattern is 0.Therefore 0 is a term of this number pattern.

(iv) If 18 is a term of this number pattern, then for some integer n, we have
$$56 - 4n = 18.$$
Then, $56 - 4n + 4n = 18 + 4n$
$$56 - 18 = 18 - 18 + 4n$$
$$38 = 4n$$
$$9\frac{1}{2} = n$$

If 18 is a term of this number pattern, the value of n should be a whole number.
Since $n = 9\frac{1}{2}$, 18 is not a term of this number pattern.

Exercise 1.1

1. Complete the table.

The general term of the number pattern	The first term when $n = 1$	The second term when $n = 2$	The third term when $n = 3$	First three terms of the number pattern
$3n + 2$	$(3 \times 1) + 2 = 5$	$(3 \times 2) + 2 = 8$	$(3 \times 3) + 2 = 11$	5, 8, 11
$5n - 1$	$(5 \times 1) - 1 = 4$	....	....	..., ..., ...
$2n + 5$	....	....	....	..., ..., ...
$20 - 2n$	....	....	....	..., ..., ...
$50 - 4n$	....	....	....	..., ..., ...
$35 - n$	....	....	....	..., ..., ...

2. The general term of a number pattern is $4n - 3$.

i. Write the first three terms of this number pattern.
ii. Find the 12th term.
iii. Which term is equal to 97?
iv. Show that 75 is not a term of this number pattern.

3. Consider the number pattern with nth term $7n + 1$.

i. Write the first three terms of this number pattern.
ii. Find the 5th term.
iii. Which term is equal to 36?
iv. Write the $(n+1)$th term, in terms of n.

4. Consider the number pattern with general term $T_n = 50 - 7n$.

i. Write the first three terms of this number pattern.
ii. Find the 10th term.
iii. Write the $(n + 1)$th term, in terms of n.
iv. Show that the terms which come after the 7th term are negative numbers.

1.2 Obtaining an expression for the general term (T_n)

In the previous section an expression was given for the general term T_n. Our objective in this section, is to obtain an expression for T_n in terms of n. Then, any term of the sequence can easily be found by using the obtained expression. Now let us consider how we can develop such an expression, through an example.
Suppose we want to find the 80th term of the sequence 5, 11, 17, 23…, which is a sequence with a common difference. That is, we want to find the value of T_{80}. First, examine the pattern given in the following table.

n	T_n	How T_n can be written in terms of n and the common difference 6.
1	5	$6 \times 1 - 1$ or $5 + 0 \times 6$
2	11	$6 \times 2 - 1$ or $5 + 1 \times 6$
3	17	$6 \times 3 - 1$ or $5 + 2 \times 6$
4	23	$6 \times 4 - 1$ or $5 + 3 \times 6$
5	29	$6 \times 5 - 1$ or $5 + 4 \times 6$

You may be wondering why the expressions $6 \times 1 - 1$, $6 \times 2 - 1$, $6 \times 3 - 1$, etc., given in the 3rd column of the table have been written. Especially, the reason why 1 is subtracted from each term may be unclear to you. This can be explained as follows.

Since the common difference of the given sequence 5, 11, 17, 23, … is 6, let us write the given sequence first and several multiples of 6 below it.

5, 11, 17, 23, 29, ...

6, 12, 18, 24, 30, ...

It is clear that the given sequence can be obtained by subtracting 1 from each multiple of 6.

That is,

the first term of the sequence = the first multiple of 6 – 1
the second term of the sequence = the second multiple of 6 –1
the third term of the sequence = the third multiple of 6 – 1

Accordingly,

the nth term of the sequence = the nth multiple of 6 – 1

$$\therefore \; T_n = 6n - 1$$

Accordingly,

$$T_{80} = 6 \times 80 - 1 = 479.$$

Therefore, the 80th term is 479.

Moreover, an expression for the general term T_n of this sequence was found above as $T_n = 6n - 1$.
We can find any term of the sequence using this expression. For example, in order to find the 24th term of this sequence, n has to be substituted with 24.

$T_{24} = 6 \times 24 - 1 = 143$

Therefore, the 24th term of the sequence is 143.

Let us consider another example.

Example 1

Given that the sequence with first four terms 15, 19, 23, 27 has a common difference, let us find an expression for the nth term.

The common difference $= 19 - 15 = 4$.
Let us write the first few terms of the given sequence, and several multiples (positive integer multiples) of 4 below them.

15, 19, 23, 27, ...
4, 8, 12, 16, ...

It is clear that the given number pattern is obtained by adding 11 to each multiple of 4.

Therefore, the expression for the general term T_n is given by $T_n = 4n + 11$.
Let us find the 100th term using this expression.

$T_{100} = 4 \times 100 + 11 = 411$

Now let us consider a sequence with a negative common difference, which therefore consists of terms which are decreasing in value.

Example 2

Let us find an expression for the general term T_n of the sequence with a common difference, of which the first four terms are 10, 7, 4, 1.

The common difference of the sequence 10, 7, 4, 1, … $= 7 - 10 = -3$.

Therefore, let us write the first few terms of the given sequence and a few multiples of –3 (integral), one below the other.

$$\begin{array}{c} 10, 7, 4, \ldots \\ -3, -6, -9, \ldots \end{array}$$

It can be seen that the terms of the sequence are obtained by adding 13 to the multiples of −3. Therefore, the general term can be written as

$$T_n = -3n + 13$$

(Or else, it can be written as $T_n = 13 - 3n$ with the positive term first.)

For example, in order to find the 30th term, $n = 30$ should be substituted in the expression for T_n.

$T_{30} = -3 \times 30 + 13 = -77$

Therefore, the 30th term is − 77.

Exercise 1.2

All sequences in this exercise have a common difference.

1. Copy the following table in your exercise book and complete it.

Pattern	**The difference between two successive terms**	**The number, whose multiples are used to develop the pattern**
5, 8, 11, 14,...	$8 - 5 = 3$	3
10, 17, 24, 31, ...		
$2\frac{1}{2}, 4, 5\frac{1}{2}, 7, \ldots$		
20, 17, 14, 11, ...		
50, 45, 40, 35, ...		
0.5, 0.8, 1.1, 1.4, ...		

2. Complete the table in relation to the number pattern 10, 17, 24, 31, ...,

Sequential order of the terms	**Term**	**How the pattern has been developed**
1st term	10	$7 \times 1 + \ldots$
2nd term	17	$7 \times 2 + \ldots$
3rd term	24	$\ldots + \ldots$
4th term	31	$\ldots + \ldots$
nth term	....	$\ldots + \ldots = \ldots$

3. Find the general term of each of the number patterns given below.

a. 1, 4, 7, 10, ...
b. 1, 7, 13, 19, ...
c. 9, 17, 25, 33, ...
d. 4, 10, 16, 22, ...
e. 22, 19, 16, 13, ...
f. 22, 20, 18, 16, ...

1.3 Solving mathematical problems involving number patterns

We can solve various mathematical problems by developing number patterns using information that is given.

Example 1

A long distance runner trains every day. On the first day he runs 500 m and on each day thereafter he runs 100 m more than on the previous day.

i. Write separately the distances he runs on the first three days.
ii. Find the general term T_n for the distance he runs on the nth day, in terms of n.
iii. Find the distance he runs on the 20th day.
iv. On which day does he run 3km?

i. The distance run on the first day = 500 m
The distance run on the second day = 500 m + 100 m = 600 m
The distance run on the third day = 500 m + 100 m + 100 m = 700 m
$\therefore$ The first three terms of the number pattern are 500, 600, 700

ii. Let us take the day as n.

The number pattern of the distance run by the athlete is built up by multiples of 100.

Therefore, the general term $T_n = 100n + 400$

iii. It is clear that the distance run on the 20th day is represented by the 20th term.

$$\begin{aligned} \text{The 20th term, } T_{20} &= (100 \times 20) + 400 \\ &= 2000 + 400 \\ &= 2400 \text{ m} \\ &= 2.4 \text{ km} \end{aligned}$$

$\therefore$ The distance run on the 20th day is 2.4 km.

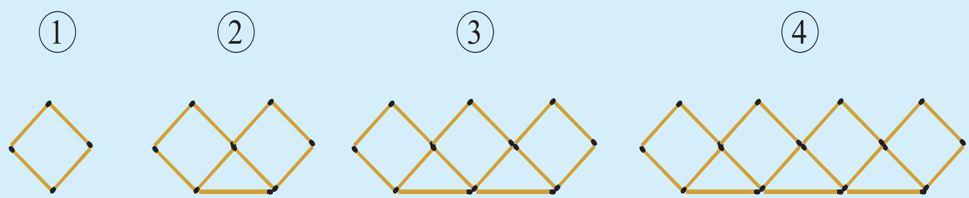
iv. Let us assume that 3000 m are run on the nth day.

$$\begin{aligned} \text{Then, } 100n + 400 &= 3000 \\ 100n + 400 - 400 &= 3000 - 400 \\ 100\,n &= 2600 \\ \therefore\ n &= \frac{2600}{100} \\ &= 26 \end{aligned}$$

Therefore, 3km are run on the 26th day.

Exercise 1.3

1. A pattern created by using matchsticks is shown below.



Complete the table in relation to the pattern given above.

Figure Number	1	2	3	4
Total number of matchsticks	....	9	....	....

i. Find the number of matchsticks needed to create the 20th figure.

ii. 219 matchsticks are required to create which figure of this pattern?

iii. Show that one matchstick remains after creating a figure of this pattern by using the maximum number of match sticks from 75 matchsticks.

2. A worker cuts pieces of rods of different lengths from iron rods which are 5 m in length, in order to build a gate by welding pieces together. The length of the shortest piece of iron rod that is cut is 15 cm. All the other pieces are cut such that the difference in length between two successive pieces which are cut is 10 cm.

i. Write the lengths of the shortest three pieces cut by the worker.
ii. Find the length of the 20th piece, when arranged in ascending order of the length, starting from the shortest piece.
iii. Show that a 5 m long rod will not be sufficient to cut the 50th piece, when arranged in ascending order of the length.

3. On the day that their school celebrated "Annual Savings Day", Yesmi and Indunil start saving money by putting Rs 100 each into their respective savings boxes. After that, they put money into their savings boxes once a week, on the same day of the week that they started saving money. Yesmi put Rs 10 and Indunil put Rs 5 each week into their respective boxes.

i. How much does Yesmi have in her savings box in the 5th week?
ii. How much does Indunil have in her savings box in the 10th week?
iii. At the end of 50 weeks, both of them open their savings boxes and check the amount that each has saved. How much more money has Yesmi saved than Indunil in the 50 weeks?

4. The seats in an outdoor stadium are arranged for a drama in 15 rows according to a pattern with a common difference, such that the first row consists of 9 seats, the second row of 12 seats, the third row of 15 seats, etc.

i. How many seats are there in total in the first five rows?
ii. How many seats are there in the 15th row?
iii. Show that the 10th row has 4 times the number of seats in the first row, according to this pattern.
iv. Which row consists of 51 seats?

Miscellaneous Exercise

1. The general terms of a few number patterns are given below.
(a) $3n - 5$ (b) $6n + 5$ (c) $6n - 5$

For each number pattern,

i. write the first three terms.
ii. find the 20th term.
iii. find the $(n - 1)$th term in terms of n.

2. Find the general term of each number pattern given below, given that each has a common difference.

i. $-3, 1, 5, 9, \ldots$ **ii.** $0, 4, 8, 12, \ldots$

iii. $1\frac{1}{2}, 2, 2\frac{1}{2}, \ldots$ **iv.** $-6, -3, 0, 3, \ldots$

3. Show that the general term of the number pattern 42, 36, 30, 24, ... with a common difference is $6\,(8 - n)$.

4. Uditha is employed in a private company. His first monthly salary is Rs 25 000. From the beginning of the second year onwards, he receives an annual salary increment of Rs 2400 per month.

i. How much is his monthly salary at the beginning of the second year?
ii. Write separately, Uditha's monthly salary during the first three years.
iii. Write an expression for his salary in the nth year in terms of n.
iv. Find Uditha's monthly salary in the 5th year, by using the expression obtained in (iii) above.

Summary

Summary

- common difference = any term except the first term – the previous term
- The general term of a sequence is denoted by T_n .
- Any term of a sequence can easily be found by using the general term.

2 Binary Numbers

By studying this lesson, you will be able to;

- identify binary numbers,
- convert a decimal number into a binary number,
- convert a binary number into a decimal number,
- add and subtract binary numbers,
- identify instances where binary numbers are used.

Introduction

Let us recall how numbers are written in the Hindu - Arabic number system, which is the number system we use.

As an example, let us consider the number 3 725. According to what we have learnt in previous grades,

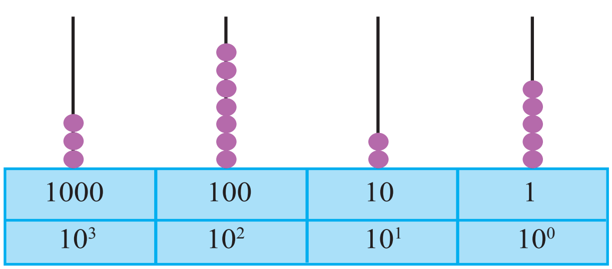
5 denotes the number of 1s (that is, the number of 10^0s),

2 denotes the number of 10s (that is, the number of 10^1s),

7 denotes the number of 100s (that is, the number of 10^2s),

3 denotes the number of 1000s (that is, the number of 10^3s).

The above can be represented on an abacus as shown below.



Observe that the number 3 725 can also be written in terms of powers of 10 as shown below.

$$3\ 725 = 3 \times 1000 + 7 \times 100 + 2 \times 10 + 5 \times 1$$

That is, $3\ 725 = 3 \times 10^3 + 7 \times 10^2 + 2 \times 10^1 + 5 \times 10^0$

If we consider 603 as another example, we can write it as shown below.

$$603 = 6 \times 10^2 + 0 \times 10^1 + 3 \times 10^0$$

In the Hindu - Arabic number system which we use, the place values are powers of ten such as 1, 10, 100 and 1000. Moreover, we use the 10 digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 to write numbers in the Hindu - Arabic number system. The method of writing numbers using these 10 digits and assigning place values which are powers of ten, is called writing the numbers in "base 10". When studying about number bases, these numbers are called "decimal numbers".

Note : $10^0 = 1$. Similarly, any nonzero base raised to the power zero is always equal to one. Accordingly, $2^0 = 1$.

2.1 Expressing numbers in the binary number system

We can use number bases other than base ten to express numbers. For example, we can express numbers in "base two" by using only the digits 0 and 1, and assigning place values which are powers of two. To do this, let us first identify several powers of two.

We can write them as;

$2^0 = 1$	$2^5 = 32$
$2^1 = 2$	$2^6 = 64$
$2^2 = 4$	$2^7 = 128$
$2^3 = 8$	$2^8 = 256$
$2^4 = 16$	$2^9 = 512$

To understand the method of writing numbers in base two, let us first consider the base ten number 13 as an example. Let us see how we can write 13 as a sum of powers of two.

The first few powers of two are;

1, 2, 4 and 8.

Using these numbers which are powers of two, we can write,

$$13 = 8 + 4 + 1$$

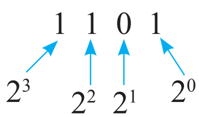
i.e., $13 = 2^3 + 2^2 + 2^0$

i.e., $13 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$.

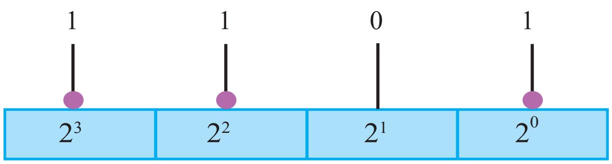
Here, we have written all the non - negative powers of two in descending order, starting from 2^3 and continuing with 2^2, 2^1 and 2^0. Also, since the power 2^3 is included, it is written as 1×2^3 and since the power 2^1 is not included, it is written as 0×2^1. Recall that we use only the digits 1 and 0 when writing base two numbers. Considering the above facts, we can write 13 as a base two number as follows.

1101

The digits 0 and 1 appearing in this base two number can be described as follows.



We can also express it using an abacus as follows.



To indicate that 1101 is a base two number, we usually write "two" as a subscript and express the number as 1101_{two}. In this lesson, whenever necessary, we indicate base ten numbers with the subscript "ten" to differentiate the base two numbers from the base ten numbers. For example, the decimal number 603 is written as 603_{ten}.

Let us consider another example. Let us write the base ten number 20_{ten} as a base two number.

By recalling the powers of two, we can write;

$$
\begin{aligned}
20 &= 16 + 4 \\
&= 2^4 + 2^2 \\
&= 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 0 \times 2^0.
\end{aligned}
$$

Hence,

$20_{ten} = 10100_{two}$.

There is an important fact to remember here. There is only one way of writing any number as a sum of distinct descending powers of two. For example, $20 = 16 + 4$ can only be written as $2^4 + 2^2$ as a sum of distinct descending powers of two. There is no other way. You can see this for yourself by attempting to find a different way. Moreover, any number can be written as a sum of powers of two. You can verify this too by writing different decimal numbers as a sum of distinct powers of two.

The above method of writing a decimal number as a sum of distinct descending powers of two, cannot be considered as a precise method. The reason for this is because it is difficult to decide what powers of two add up to a number, when the number is large. For example, it is not easy to determine the powers of two that add up to the decimal number 3905_{ten}.

Therefore, let us now consider another method that can be used to convert any decimal number to a binary number fairly easily.

Consider 22_{ten} as an example. To write this as a binary number, we need to first divide 22 by 2 and write the remainder also.

$$\begin{array}{r|l} 2 & \underline{22} \\ & 11 \quad \text{remainder } 0 \end{array}$$

Next we need to divide the quotient 11 which we obtained by dividing 22 by 2, again by 2.

$$\begin{array}{r|ll} 2 & \underline{22} & \\ 2 & \underline{11} & \text{remainder } \ 0 \\ & \ \ 5 & \text{remainder } \ 1 \end{array}$$

We need to continue dividing the quotient by 2 and noting down the remainder, until we get 0 as the quotient and 1 as the remainder. The complete division is shown below.

$$\begin{array}{r|rll} 2 & \underline{22} & & \\ 2 & \underline{11} & \text{remainder} & 0 \\ 2 & \underline{5} & \text{remainder} & 1 \\ 2 & \underline{2} & \text{remainder} & 1 \\ 2 & \underline{1} & \text{remainder} & 0 \\ & 0 & \text{remainder} & 1 \end{array}$$

$$1 \ 0 \ 1 \ 1 \ 0_{two}$$

When the highlighted remainders are written from bottom to top, we obtain the required base two number.

$$22_{ten} = 10110_{two}$$

Let us see whether we can verify this answer using the method we discussed earlier of writing the number as a sum of powers of two.

$$22 = 16 + 4 + 2 = 2^4 + 2^2 + 2^1 = 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$$
$$= 10110_{two}$$

The answer is verified.

Example 1

Write each decimal number given below as a binary number.

i. 32_{ten}

2	32	
2	16	0
2	8	0
2	4	0
2	2	0
2	1	0
	0	1

$32_{ten} = 100000_{two}$

ii. 154_{ten}

2	154	
2	77	0
2	38	1
2	19	0
2	9	1
2	4	1
2	2	0
2	1	0
	0	1

$154_{ten} = 10011010_{two}$

Exercise 2.1

Convert the decimal numbers (base ten numbers) given below into binary numbers (base two numbers).

a. 4	**b.** 9	**c.** 16	**d.** 20	**e.** 29
f. 35	**g.** 43	**h.** 52	**i.** 97	**j.** 168

2.2 Converting binary numbers into decimal numbers

Decimal numbers were converted into binary numbers in the above section 2.1. In this section we consider the inverse process; that is, converting binary numbers into decimal numbers. This can be done fairly easily. Let us learn how to do this by considering an example.

In section 2.1, when we wrote the decimal number 13 as a binary number, we obtained 1101_{two}. Let us recall what the digits 1, 1, 0 and 1 represent.

1	1	0	1
number of 2^3s	number of 2^2s	number of 2^1s	number of 2^0s

Therefore, by adding all the values of the powers of two in 1101_{two} we get the corresponding decimal representation.

$$1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 1 \times 8 + 1 \times 4 + 0 \times 2 + 1 \times 1$$
$$= 8 + 4 + 1 = 13.$$

By simplifying, we obtain the corresponding decimal number 13.

Example 1

Write 101100_{two} as a decimal number.

First, it should be noted that the place value of the leftmost digit of this base two number is 2^5 and that the other place values are obtained by reducing the index by one (starting from 5) for each move from left to right. Then the required decimal number can be found by multiplying each power of two (place value) by the relevant co-efficient and adding all the terms together.

$$\begin{aligned} 101100_{two} &= 1\times 2^5 + 0\times 2^4 + 1\times 2^3 + 1\times 2^2 + 0\times 2^1 + 0\times 2^0 \\ &= 2^5 + 2^3 + 2^2 = 32 + 8 + 4 \\ &= 44_{ten} \end{aligned}$$

Therefore, when 101100_{two} is written in base 10 we obtain 44_{ten}.

Note: This answer can be verified by converting 44_{ten} back into a binary number.

Exercise 2.2

Convert the binary numbers given below into base ten numbers (decimal numbers).

a. 101_{two} **b.** 1101_{two} **c.** 1011_{two} **d.** 1100_{two} **e.** 11111_{two}

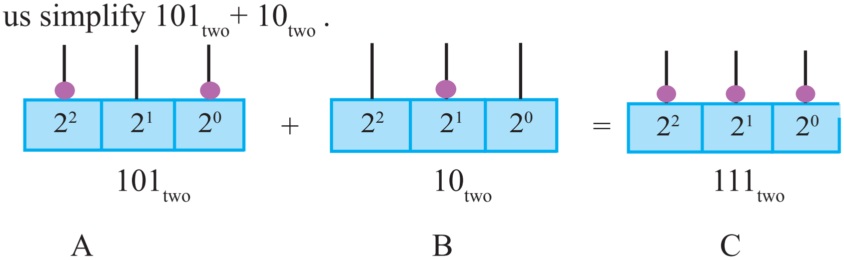
f. 100111_{two} **g.** 110111_{two} **h.** 111000_{two} **i.** 111110_{two} **j.** 110001_{two}

2.3 Adding binary numbers

When representing binary numbers on an abacus, the maximum number of counters that can be placed on a rod is 1. Moreover, instead of placing two counters on a rod, we always place one counter on the rod to the left of it.

Let us learn how to add binary numbers with the aid of two abacuses.

Let us simplify $101_{two} + 10_{two}$.



Let us represent the sum of the numbers represented on the abacuses A and B on the abacus C.
When we consider the two abacuses A and B;

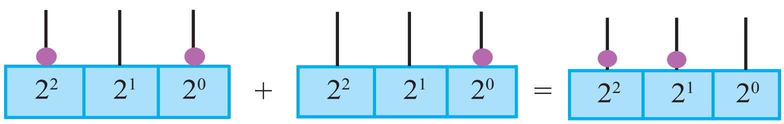
the sum of the counters on the 2^0 rods is 1,

the sum of the counters on the 2^1 rods is 1,

the sum of the counters on the 2^2 rods is 1.

Therefore, $101_{two} + 10_{two} = 111_{two}$

Now, let us obtain the value of $101_{two} + 1_{two}$ using the abacuses.



A B C

The counter on the 2^0 rod in A and the counter on the 2^0 rod in B, cannot both be placed on the 2^0 rod in C, because there cannot be two counters on any rod of an abacus used to represent a binary number. Instead, one counter should be placed on the rod to the left of the 2^0 rod. This is shown on the rod 2^1 in C.

Therefore, $101_{two} + 1_{two} = 110_{two}$.

This is clarified further by adding the numbers vertically.

$$\begin{array}{r} 101_{two} \\ +\quad 1_{two} \\ \hline 110_{two} \\ \hline\hline \end{array}$$

Adding from right to left; first, one 2^0s + one 2^0s = one 2^1s and zero 2^0s.

Then, one 2^1s + zero 2^1s = one 2^1s. Finally, one 2^2s + zero 2^2s = one 2^2s.

Example 1

Find the value.

(i) $11101_{two} + 1101_{two}$ (ii) $1110_{two} + 111_{two}$

(i)
$$\begin{array}{r} {\scriptstyle 11\ \ 1\ \ } \\ 11101_{two} \\ +\ 1101_{two} \\ \hline 101010_{two} \\ \hline\hline \end{array}$$

(ii)
$$\begin{array}{r} {\scriptstyle 11\ \ \ \ \ } \\ 1110_{two} \\ +\ 111_{two} \\ \hline 10101_{two} \\ \hline\hline \end{array}$$

Note: When adding binary numbers observe the relationships given below.

$$1_{two} + 0_{two} = 1_{two}$$
$$1_{two} + 1_{two} = 10_{two}$$
$$1_{two} + 1_{two} + 1_{two} = 11_{two}$$

Exercise 2.3

1. Find the value.

a.
$$\begin{array}{r} 111_{two} \\ +101_{two} \\ \hline \end{array}$$

b.
$$\begin{array}{r} 10111_{two} \\ +\ 1011_{two} \\ \hline \end{array}$$

c.
$$\begin{array}{r} 1011_{two} \\ +\ 11101_{two} \\ \hline \end{array}$$

d. $11101_{two} + 1110_{two}$ **e.** $11011_{two} + 11_{two}$ **f.** $100111_{two} + 11_{two} + 1_{two}$

g. $11_{two} + 111_{two} + 1111_{two}$ **h.** $11110_{two} + 1110_{two} + 110_{two}$

2. Fill each cage with the suitable digit.

a.
$$\begin{array}{r} 11_{two} \\ +1\square_{two} \\ \hline 1\square 1_{two} \\ \hline\hline \end{array}$$

b.
$$\begin{array}{r} 1\ 10\ \square_{two} \\ +\ \square 1\ 1_{two} \\ \hline 1\square 100_{two} \\ \hline\hline \end{array}$$

c.
$$\begin{array}{r} 10\ 0\ 1_{two} \\ +\ \ \square 1\ \square_{two} \\ \hline \square 00\square 0_{two} \\ \hline\hline \end{array}$$

d.
$$\begin{array}{r} 1\ 1\ 1\ 0_{two} \\ +\ \ 1\square\square_{two} \\ \hline 10\square 0\ 1_{two} \\ \hline\hline \end{array}$$

e.
$$\begin{array}{r} 1\ \square 1\ \square_{two} \\ +\ \ 1\ \square 1_{two} \\ \hline 1\square\ 000_{two} \\ \hline\hline \end{array}$$

f.
$$\begin{array}{r} 1\ 1\ \square 1_{two} \\ +1\ 1\ 1\ 0_{two} \\ \hline 1\square\ \square\ 1\square_{two} \\ \hline\hline \end{array}$$

2.4 Subtracting binary numbers

When adding binary numbers, we saw that whenever we obtained a sum of 2 in a particular position, we replaced it with 1 in the position left of it.

$$\begin{array}{r} 101_{two} \\ +\quad 1_{two} \\ \hline 110_{two} \\ \hline\hline \end{array} \qquad \text{(right hand column: } 1_{two} + 1_{two} = 10_{two}\text{)}$$

Now let us find the value of $110_{two} - 1_{two}$. According to the above addition, the answer should be 101_{two} . Let us consider how this answer is obtained.

$$\begin{array}{cccc} & 2^2 & 2^1 & 2^0 \\ & 1 & 1 & 0_{two} \\ - & & & 1_{two} \\ \hline & 1 & 0 & 1_{two} \\ \hline\hline \end{array}$$

We cannot subtract 1 from 0 in the rightmost column. Therefore, let us take 1 from the 2^1 column which is to the left of it.The value of this is 2 in the 2^0 column. Now we subtract 1 from this 2 to obtain 1 in the rightmost column. Now there is 0 instead of 1 in the column 2^1.

Therefore, $110_{two} - 1_{two} = 101_{two}$.

Example 1

$$\begin{array}{r} 1101_{two} \\ -\quad 111_{two} \\ \hline 110_{two} \\ \hline\hline \end{array}$$

Let us check the accuracy of the answer by considering $110_{two} + 111_{two}$.

$$110_{two} + 111_{two} = \underline{\underline{1101}}_{\ two}$$

Note: It is very important to develop the habit of checking the accuracy of an answer to a subtraction problem using addition as shown above.

Exercise 2.4

1. Find the value.

a. $\begin{array}{r} 11_{two} \\ -\ \ 1_{two} \\ \hline \end{array}$ **b.** $\begin{array}{r} 10_{two} \\ -\ \ 1_{two} \\ \hline \end{array}$ **c.** $\begin{array}{r} 101_{two} \\ -\ \ 1_{two} \\ \hline \end{array}$ **d.** $\begin{array}{r} 101_{two} \\ -\ \ 11_{two} \\ \hline \end{array}$

e. $111_{two} - 11_{two}$ **f.** $110_{two} - 11_{two}$ **g.** $1100_{two} - 111_{two}$ **h.** $\begin{array}{r} 10001_{two} \\ -\ \ 111_{two} \\ \hline \end{array}$

i. $\begin{array}{r} 100000_{two} \\ -\ 11011_{two} \\ \hline \end{array}$ **j.** $\begin{array}{r} 100011_{two} \\ -\ 10001_{two} \\ \hline \end{array}$ **k.** $11000_{two} - 1111_{two}$ **l.** $101010_{two} - 10101_{two}$

2.5 Applications of binary numbers

The fundamental digits in the binary number system are 0 and 1. Many modern digital instruments are made based on this feature. When designing lighting system circuits, "current off" and "current on" conditions are represented by 0 and 1.

If ⊛ is used to represent the current "on" condition and ○ to represent the current "off" condition, then the combination ⊛ ○ ○ ⊛ is represented by 1001_{two}. The storing of data and computations done in computers and calculators are based on this concept. Any number system can be developed under the same principles used to develop the binary number system. Storing of data can be done using other number systems too.

Note: If a number system is developed using base four, only the fundamental digits 0, 1, 2 and 3 are used.
For example, the decimal number 4 is expressed as 10_{four} in this number system.
In the base five number system, the fundamental digits are 0, 1, 2, 3 and 4, and the decimal number 5 is expressed as 10_{five} in this system.

Miscellaneous Exercise

1. Find the value.

a. $1101_{two} + 111_{two} - 1011_{two}$

b. $11111_{two} - (101_{two} + 11_{two})$

c. $110011_{two} - 1100_{two} - 110_{two}$

2. Write the next number, after adding 1 to each given number. 1_{two} , 11_{two}, 111_{two}, 1111_{two}, 11111_{two}, 111111_{two}

3. Represent the decimal number 4^2 as a binary number.

4. i. Simplify $49_{ten} - 32_{ten}$ and convert the answer into a binary number.
ii. Convert 49_{ten} and 32_{ten} into binary numbers and find their difference. See whether the answer you obtain is the same as the answer in (1) above.

Summary

Summary

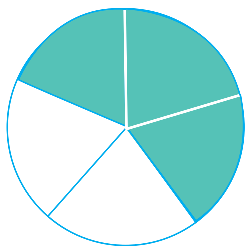
- In the binary number system, the fundamental digits are 0 and 1.
- The place values of the binary number system are; 2^0, 2^1, 2^2, 2^3, 2^4, 2^5, 2^6, etc.

3 Fractions

By studying this lesson, you will be able to;
- simplify expressions of fractions which contain "of",
- simplify expressions of fractions which contain brackets,
- identify the BODMAS method and solve problems involving fractions.

Fractions

Let us recall the facts that were learnt about fractions in previous grades. The circle shown below is divided into five equal parts of which three are shaded.

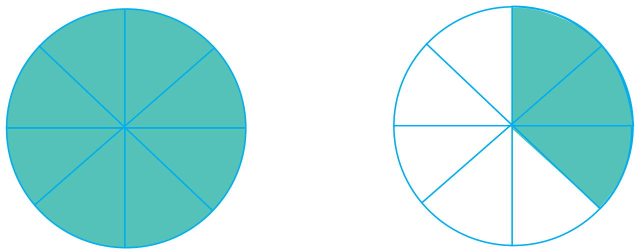


The shaded region can be expressed as $\frac{3}{5}$ of the whole region. We can express this in terms of the area of the circle too. That is, the shaded area is $\frac{3}{5}$ of the area of the whole figure. If the total area of the circle is taken as 1 unit, then the shaded area is $\frac{3}{5}$ units.

When an object is divided into equal portions, one portion or several portions can be expressed as a fraction. A portion of a collection can also be expressed as a fraction.
For example, if we consider a team consisting of three boys and two girls, then the boys in the team can be considered as $\frac{3}{5}$ of the team. Here, if the whole team is considered as a unit, then the boys in the team can be expressed as $\frac{3}{5}$.

You have learnt that fractions between zero and one such as $\frac{3}{4}$, $\frac{1}{2}$ and $\frac{2}{3}$ are called proper fractions.

Let us now recall the facts that have been learnt previously about mixed numbers and improper fractions.
Two identical circles are given below. One is shaded completely and three parts of the other (which is divided into equal parts) are shaded.



If a circle is considered as one unit, the shaded fraction is $1 + \frac{3}{8}$. This is usually written as $1\frac{3}{8}$, which is called a mixed number ("mixed fractions" are most often called "mixed numbers"). This can also be written as $\frac{11}{8}$, which is called an improper fraction. It is important to remember here that the mixed number and the improper fraction are expressed by taking a circle as one unit.

Some other examples of mixed numbers are $1\frac{1}{2}$, $3\frac{2}{5}$, $2\frac{3}{7}$.

$\frac{3}{2}$, $\frac{8}{5}$, $\frac{11}{4}$ are examples of improper fractions. Fractions such as $\frac{3}{3}$, $\frac{5}{5}$, $\frac{1}{1}$ which are equal to 1 are also considered as improper fractions.

You have learnt to represent mixed numbers as improper fractions and improper fractions as mixed numbers.

Accordingly,

(i) $1\frac{1}{2} = \frac{3}{2}$ and

(ii) $\frac{5}{3} = 1\frac{2}{3}$.

We can obtain a fraction equivalent to a given fraction by multiplying or dividing both the denominator and the numerator by the same number (which is not zero).

For example,

$$\frac{2}{5} = \frac{2 \times 2}{5 \times 2} = \frac{4}{10}$$

$$\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}$$

The addition and subtraction of fractions can be performed easily if the denominators of the fractions are equal.
For example,

(i) $\frac{1}{5} + \frac{4}{5} - \frac{2}{5}$

$$\frac{1}{5} + \frac{4}{5} - \frac{2}{5} = \frac{1 + 4 - 2}{5}$$

$$= \frac{3}{5}$$

If the denominators of the fractions are unequal, then we convert the fractions into equivalent fractions with equal denominators.

For example,

(ii) $\frac{1}{4} + \frac{2}{3} - \frac{5}{6} = \frac{1 \times 3}{4 \times 3} + \frac{2 \times 4}{3 \times 4} - \frac{5 \times 2}{6 \times 2}$

$$= \frac{3}{12} + \frac{8}{12} - \frac{10}{12}$$

$$= \frac{3 + 8 - 10}{12}$$

$$= \frac{1}{12}$$

- When multiplying two fractions, the numerator of the product is obtained by multiplying the numerators of the two fractions and the denominator is obtained by multiplying the denominators of the two fractions.

Example 1

$\frac{2}{5} \times \frac{1}{3}$

$$\frac{2}{5} \times \frac{1}{3} = \frac{2 \times 1}{5 \times 3} = \underline{\underline{\frac{2}{15}}}$$

Example 2

$1\frac{1}{3} \times 1\frac{3}{4}$

$$1\frac{1}{3} \times 1\frac{3}{4} = \frac{\cancel{4}^{1}}{3} \times \frac{7}{\cancel{4}_{1}} \quad \text{(converting the mixed numbers into improper fractions)}$$

$$= \frac{7}{3}$$

$$= \underline{\underline{2\frac{1}{3}}}$$

- If the product of two numbers is 1, then each number is said to be the reciprocal of the other.

Accordingly, since $2 \times \frac{1}{2} = 1$,

2 is the reciprocal of $\frac{1}{2}$ and $\frac{1}{2}$ is the reciprocal of 2.

You have learnt that the reciprocal of a number can be obtained by interchanging the denominator and the numerator.

Hence, the reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$ (In the same way, the reciprocal of $\frac{b}{a}$ is $\frac{a}{b}$.)

- In grade 8 you learnt that dividing a number by another number means multiplying the first number by the reciprocal of the second number.

Let us revise this by considering a couple of examples.

Example 3

$\frac{4}{3} \div 2$

$$\frac{4}{3} \div 2 = \frac{{}^{2}\cancel{4}}{3} \times \frac{1}{\cancel{2}_{1}}$$

$$= \underline{\underline{\frac{2}{3}}}$$

Example 4

$1\frac{2}{7} \div 1\frac{1}{2}$

$$1\frac{2}{7} \div 1\frac{1}{2} = \frac{9}{7} \div \frac{3}{2}$$

$$= \frac{{}^{3}\cancel{9}}{7} \times \frac{2}{\cancel{3}_{1}}$$

$$= \underline{\underline{\frac{6}{7}}}$$

Do the following review exercise to revise what you have learnt thus far about fractions.

Review Exercise

1. For each of the fractions given below, write two equivalent fractions.

i. $\frac{2}{3}$ ii. $\frac{4}{5}$ iii. $\frac{4}{8}$ iv. $\frac{16}{24}$

2. Express each mixed number given below as an improper fraction.

i. $1\frac{1}{2}$ ii. $2\frac{3}{4}$ iii. $3\frac{2}{5}$ iv. $5\frac{7}{10}$

3. Express each improper fraction given below as a mixed number.

i. $\frac{7}{3}$ ii. $\frac{19}{4}$ iii. $\frac{43}{4}$ iv. $\frac{36}{7}$

4. Find the value.

i. $\frac{3}{7}+\frac{2}{7}$ ii. $\frac{5}{6}-\frac{2}{3}$ iii. $\frac{7}{12}+\frac{3}{4}-\frac{2}{3}$

iv. $1\frac{1}{2}+2\frac{1}{4}$ v. $3\frac{5}{6}-1\frac{2}{3}$ vi. $1\frac{1}{2}+2\frac{1}{4}-1\frac{2}{3}$

5. Simplify.

i. $\frac{1}{2}\times\frac{4}{7}$ ii. $\frac{2}{3}\times\frac{5}{8}\times\frac{3}{10}$ iii. $1\frac{3}{5}\times 2\frac{1}{2}$ iv. $3\frac{3}{10}\times 2\frac{1}{3}\times 4\frac{2}{7}$

6. Write the reciprocal of each of the following.

i. $\frac{1}{3}$ ii. $\frac{1}{7}$ iii. $\frac{3}{8}$ iv. 5 v. $2\frac{3}{5}$

7. Simplify.

i. $\frac{6}{7}\div 3$ ii. $8\div\frac{4}{5}$ iii. $\frac{9}{28}\div\frac{3}{7}$ iv. $5\frac{1}{5}\div\frac{6}{7}$ v. $1\frac{1}{2}\div 2\frac{1}{4}$

3.1 Simplifying expressions of fractions containing "of"

We know that $\frac{1}{2}$ of 100 rupees is 50 rupees.

We also know that this is one half of 100 rupees and that its value can be obtained by dividing 100 by 2.

This can be written as $100 \div 2$.

That is, $100 \times \frac{1}{2}$ (multiplying by the reciprocal).

Accordingly, $\frac{1}{2}$ of 100 $= 100 \times \frac{1}{2} = \frac{1}{\cancel{2}} \times \cancel{100} = 50$.

According to the above facts, $\frac{1}{2}$ of 100 can be written as $\frac{1}{2} \times 100$.

Let us similarly determine how much $\frac{1}{5}$ of 20 kilogrammes is.

This amount can be considered as one part from 5 equal parts into which 20 kilogrammes is divided.

We can write this as $20 \div 5$.

That is, $20 \times \frac{1}{5}$ (multiplying by the reciprocal).

$20 \times \frac{1}{5} = \frac{1}{\cancel{5}} \times \cancel{20}^{4} = 4.$

According to the above facts, $\frac{1}{5}$ of 20 can be written as $\frac{1}{5} \times 20$.

It can be seen from the above instances that we can replace the term "of" by the operation "×".

$\frac{1}{2}$ of 100 rupees $= \frac{1}{2} \times 100$ rupees

$\frac{1}{5}$ of 20 kilogrammes $= \frac{1}{5} \times 20$ kilogrammes

Now let us find the value of $\frac{1}{2}$ of $\frac{1}{3}$. Let us illustrate this using figures.

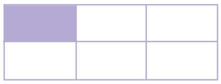
When a unit is divided into three equal parts, one part is $\frac{1}{3}$.

 If this figure is taken as one unit, $\frac{1}{3}$ of it is shown below.

$\frac{1}{3}$

Let us separate out $\frac{1}{2}$ of the shaded region.

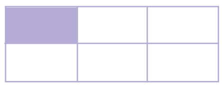
$\frac{1}{2}$



Accordingly,

$\frac{1}{3}$

$\frac{1}{2}$ of $\frac{1}{3} = \frac{1}{6}$



According to the figure, it is clear that $\frac{1}{2}$ of $\frac{1}{3}$ is $\frac{1}{6}$.

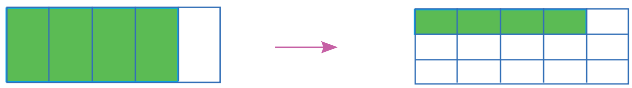
More accurately, if $\frac{1}{3}$ of a unit is taken and then $\frac{1}{2}$ of that $\frac{1}{3}$ is separated out, the portion we get is $\frac{1}{6}$ of the original unit.

Moreover, based on what we have learnt regarding multiplying fractions, we obtain $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$.

Accordingly, we can express $\frac{1}{2}$ of $\frac{1}{3}$ as $\frac{1}{2} \times \frac{1}{3}$.

Let us consider another example to verify this. Let us simplify $\frac{1}{3}$ of $\frac{4}{5}$.

Let us consider the rectangle given below as one unit.



According to the figure, it is clear that $\frac{1}{3}$ of $\frac{4}{5}$ is $\frac{4}{15}$.

Moreover, $\frac{1}{3} \times \frac{4}{5} = \frac{4}{15}$.

Therefore we can write, $\frac{1}{3}$ of $\frac{4}{5} = \frac{1}{3} \times \frac{4}{5}$.

It is clear that , we can replace "of" by the mathematical operation "multiplication" in the expressions $\frac{1}{2}$ of $\frac{1}{3}$ and $\frac{1}{3}$ of $\frac{4}{5}$.

Example 1

Find the value of $\frac{1}{2}$ of $\frac{2}{3}$.

$$\frac{1}{2} \text{ of } \frac{2}{3} = \frac{1}{2} \times \frac{2}{3} \quad \text{(writing } \times \text{ for "of")}$$

$$= \underline{\underline{\frac{1}{3}}}$$

Example 2

How much is $\frac{2}{3}$ of $1\frac{4}{5}$?

$$\frac{2}{3} \text{ of } 1\frac{4}{5} = \frac{2}{3} \times \frac{9}{5}$$

$$= \frac{6}{5}$$

$$= \underline{\underline{1\frac{1}{5}}}$$

Example 3

How much is $\frac{3}{5}$ of 500 metres?

$$\frac{3}{5} \text{ of } 500 = \frac{3}{5} \times 500$$

$$= \underline{\underline{300 \text{ m}}}$$

Exercise 3.1

1. Simplify the following expressions.

i. $\frac{2}{3}$ of $\frac{4}{5}$ **ii.** $\frac{6}{7}$ of $\frac{1}{3}$ **iii.** $\frac{2}{5}$ of $\frac{5}{8}$ **iv.** $\frac{5}{6}$ of $\frac{9}{11}$

v. $\frac{2}{7}$ of $1\frac{3}{4}$ **vi.** $1\frac{1}{3}$ of $2\frac{5}{8}$ **vii.** $1\frac{3}{11}$ of $5\frac{1}{2}$ **viii.** $\frac{5}{9}$ of $1\frac{4}{5}$

2. Find the value.

i. How much is $\frac{3}{4}$ of 64 rupees?

ii. How many grammes is $\frac{2}{5}$ of 400 g?

iii. How many hectares is $\frac{1}{3}$ of 6 ha?

iv. How many metres is $\frac{1}{8}$ of 1 km?

3. A person who owns $\frac{3}{5}$ of a land, gives $\frac{1}{3}$ of it to his daughter. What is the portion received by the daughter as a fraction of the whole land?

4. Nimal's monthly income is 40 000 rupees. He spends $\frac{1}{8}$ of this on travelling. How much does he spend on travelling?

3.2 Simplifying expressions with brackets according to the BODMAS rule

A numerical expression (or algebraic expression) may involve several of the operations addition, subtraction, division, multiplication and raising to the power of. There should be agreement on the order in which these operations should be performed and a set of rules describing it. In previous grades we learnt these rules to some extent. In this section we will discuss the "BODMAS" rule that is used when simplifying fractions.

The acronym "BODMAS" stands for brackets, orders/of, division, multiplication, addition and subtraction. When simplify numerical expressions, priority is given according to the BODMAS order. However, some operations have the same priority. Multiplication and division have equal priority and so do addition and subtraction. Accordingly, expressions should be simplified as follows.

1. First, simplify all expressions within brackets.

2. Then simplify powers and roots (expressions with indices) and the expressions with "of ".

* Simplifying expressions with powers and roots is not included in the syllabus.

3. Next, perform divisions and multiplications. These have equal priority and hence if both these operations are involved, priority is given from left to right.
4. Finally perform addition and subtraction. Since these too have equal priority, precedence is given from left to right, as in 3 above.

The BODMAS rule can be used to simplify expressions with fractions too. In some expressions of fractions the term "of" is used.
For example,

$$\frac{5}{12} \text{ of } \frac{6}{25}$$

As learnt in the previous section, this means $\frac{5}{12} \times \frac{6}{25}$.

A consensus is needed on how a fairly complex expression such as $\frac{2}{3} \div \frac{6}{25}$ of $\frac{5}{12} \times \frac{1}{2}$ is to be simplified. Here, precedence is given to "of" over $\div$ and $\times$.

Note: Since "of" and "raising to the power of" have the same priority, the "O" in BODMAS is considered to stand for both "of" and "order". However in this syllabus only "of" is considered.

Now, let us consider how the BODMAS rule is used in simplifying the expression $\frac{1}{4} + \frac{5}{6} \times \frac{1}{2} \div \frac{4}{3}$ of $\frac{3}{2}$.

$\frac{1}{4} + \frac{5}{6} \times \frac{1}{2} \div \frac{4}{3} \text{ of } \frac{3}{2} = \frac{1}{4} + \frac{5}{6} \times \frac{1}{2} \div \left(\frac{4}{3} \times \frac{3}{2}\right)$ (inserting brackets after replacing "of" by "×" , to indicate that this operation should be performed first)

$$= \frac{1}{4} + \frac{5}{6} \times \frac{1}{2} \div 2$$

$= \frac{1}{4} + \left(\frac{5}{6} \times \frac{1}{2}\right) \div 2$ (inserting brackets to indicate the mathematical operation which is to be performed next)

$= \frac{1}{4} + \frac{5}{12} \times \frac{1}{2}$ (multiplying by $\frac{1}{2}$ instead of dividing by 2)

$= \frac{1}{4} + \left(\frac{5}{12} \times \frac{1}{2}\right)$ (inserting brackets to indicate the mathematical operation which is to be performed next)

$= \frac{1}{4} + \frac{5}{24}$

$= \frac{6}{24} + \frac{5}{24}$ (writing both fractions with a common denominator)

$= \frac{11}{24}$

Note: The order in which the mathematical operations in an expression should be performed can be indicated very easily using brackets.

Consider the following expression.

$$\frac{5}{4} \times \frac{3}{4} - \frac{1}{3} \text{ of } \frac{1}{5} \div \frac{2}{3} \div \frac{8}{9}$$

How this should be simplified according to the BODMAS rule can be expressed using brackets as follows.

$$\left(\frac{5}{4} \times \frac{3}{4}\right) - \left(\left(\left(\frac{1}{3} \text{ of } \frac{1}{5}\right) \div \frac{2}{3}\right) \div \frac{8}{9}\right)$$

There are disadvantages in using brackets too. When brackets are used, the expression will seem long and complex. Moreover, when simplifying an expression with the aid of a calculator, we have to insert brackets carefully because there is a greater chance of making an error. Therefore, it is important to decide on a convention to simplify expressions without using brackets. Such a convention is necessary especially when writing software for computers and calculators. However, a common convention accepted worldwide has not been agreed upon yet. There are several conventions which are accepted by different countries. Similarly, manufacturers of computers and calculators also have their own conventions.

Now let us consider some examples of expressions with fractions which are simplified using the BODMAS convention.

Example 1

Simplify the expression $\frac{4}{10}$ of $\left(\frac{1}{6}+\frac{1}{4}\right)$ and write the answer in the simplest form.

$$\frac{4}{10} \text{ of } \left(\frac{1}{6}+\frac{1}{4}\right) = \frac{4}{10} \times \left(\frac{2}{12}+\frac{3}{12}\right)$$

$$= \frac{4}{10} \times \frac{5}{12} = \underline{\underline{\frac{1}{6}}}$$

Example 2

Simplify $\left(1\frac{2}{5} \div 2\frac{1}{3}\right)$ of $\left(\frac{2}{3}-\frac{1}{2}\right)$.

$$\left(1\frac{2}{5} \div 2\frac{1}{3}\right) \text{ of } \left(\frac{2}{3}-\frac{1}{2}\right) = \left(\frac{7}{5} \div \frac{7}{3}\right) \text{ of } \left(\frac{4}{6}-\frac{3}{6}\right)$$

$$= \left(\frac{7}{5} \times \frac{3}{7}\right) \text{ of } \frac{1}{6}$$

$$= \frac{3}{5} \times \frac{1}{6}$$

$$= \underline{\underline{\frac{1}{10}}}$$

Exercise 3.2

1. Simplify and write the answer in the simplest form.

i. $\frac{1}{2}+\frac{2}{3}\times\frac{5}{6}$ **ii.** $\frac{1}{4}$ of $3\frac{1}{3}\div 2\frac{1}{6}$ **iii.** $\frac{3}{5}\times\left(\frac{1}{3}+\frac{1}{2}\right)$

iv. $\frac{1}{4}$ of $\left(3\frac{1}{3}\div 2\frac{1}{6}\right)$ **v.** $3\frac{3}{4}\div\left(2\frac{1}{2}+3\frac{1}{4}\right)$ **vi.** $\left(1\frac{2}{3}\times\frac{3}{5}\right)+\left(\frac{3}{4}+\frac{1}{2}\right)$

vii. $2\frac{2}{3}\times\left(1\frac{1}{4}-\frac{1}{12}\right)\div 2\frac{1}{3}$ **viii.** $\frac{5}{6}\div\frac{7}{18}$ of $\frac{2}{3}\times\frac{3}{4}$

2. A person puts aside $\frac{1}{4}$ of his income for food and $\frac{1}{2}$ for his business and saves the remaining amount. What fraction of his income does he save?

3. Kumuduni walked $\frac{1}{8}$ of a journey, travelled $\frac{2}{3}$ of it by train and travelled the remaining distance by bus.

(i). Express the distance she travelled by foot and by train as a fraction of the total distance.

(ii). Express the distance she travelled by bus as a fraction of the total distance.

4. A father gave $\frac{1}{2}$ of his land to his son and $\frac{1}{3}$ to his daughter. The son donated $\frac{1}{5}$ of his portion and the daughter $\frac{2}{5}$ of her portion to a charitable foundation. The foundation decided to construct a building on half the land it received. On what fraction of the total land was the building constructed?

For further knowledge

This is only for your knowledge and will not be checked in exams.
Let us consider the expression

$$8 - 3 \times (4 + 1) + 12 \div 3 \times 3^2 \div 4 \text{ as an example.}$$

How the above expression is simplified using the BODMAS rule is described below.

- First the expression 4 +1 which is within brackets is simplified. This is equal to 5. Therefore we obtain

$$8 - 3 \times 5 + 12 \div 3 \times 3^2 \div 4$$

- Next the power 3^2 is simplified. This is equal to 9. Hence we obtain

$$8 - 3 \times 5 + 12 \div 3 \times 9 \div 4$$

- Next the multiplications and divisions are performed from left to right. Therefore, 3×5 is simplified first. This is equal to 15. Therefore we obtain

$$8 - 15 + 12 \div 3 \times 9 \div 4$$

- Next, $12 \div 3$ is simplified. This is equal to 4. Hence we obtain

$$8 - 15 + 4 \times 9 \div 4$$

- Next, 4×9 is simplified. This is equal to 36. Now the expression is

$$8 - 15 + 36 \div 4$$

- Then $36 \div 4$ is simplified. This is equal to 9. Therefore we obtain

$$8 - 15 + 9$$

- Now, since addition and subtraction have equal priority, simplification is done from left to right. Therefore we obtain

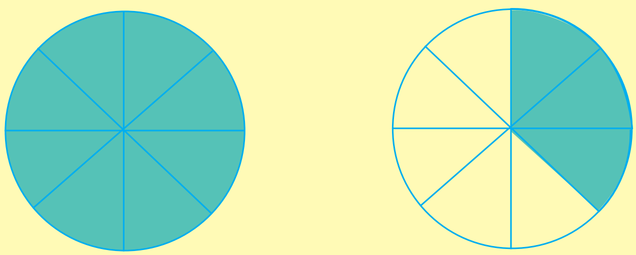
$$-7 + 9$$

- Finally we get $-7 + 9 = 2$ as the answer. According to the BODMAS order,

$$8 - 3 \times (4 + 1) + 12 \div 3 \times 3^2 \div 4 = 2.$$

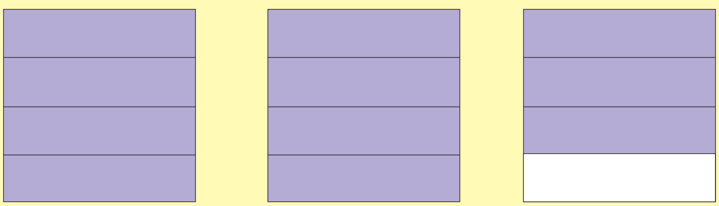
For further knowledge

This is only for your knowledge and will not be checked in exams.
Recall the figure on page 28.



We know that the shaded fraction is $1\frac{3}{8}$, if a circle is considered as one unit. This can be written as $\frac{11}{8}$.

If both these circles are considered as one unit, then the shaded fraction is $\frac{11}{16}$.



In the above diagram, if one square is considered as one unit, the shaded portion is

$2\frac{3}{4}$; That is, $\frac{11}{4}$.

a. What is the shaded fraction if all three squares together are considered as one unit?

b. What is the shaded fraction if half a square is considered as one unit?

Answers

a. $\frac{11}{12}$ **b.** $5\frac{1}{2}$

Summary

Summary

The order in which the mathematical operations are manipulated when simplifying fractions, is as follows:

- B - Brackets
- O - Of
- D - Division
- M - Multiplication
- A - Addition
- S - Subtraction

4 Percentages

By studying this lesson you will be able to;

- calculate the profit earned or loss incurred through a sale,
- calculate the profit percentage or loss percentage,
- identify what commissions and discounts are,
- perform calculations in relation to commissions and discounts.

4.1 Profit and Loss

Most of the items that we use in our day to day lives are bought from supermarkets. People who sell these items are known as sellers whereas people who buy them are known as customers.

The goods sold by sellers are either produced by them or are bought from someone else. In producing or buying goods, a cost is incurred. An item produced or purchased at a cost is generally sold at a price which is greater than the incurred cost. When selling goods at a price which is greater than the cost, it is said that the seller has earned a **profit** from the sale.

A seller will not always be able to sell his goods at a profit. For example, when goods are damaged or about to expire, they may have to be sold at a price which is less than the cost. In such a situation, it is said that the seller has incurred **a loss**. When a seller sells an item at the price at which he bought it, he neither earns a profit nor incurs a loss.

Accordingly, if
the selling price > the cost,
then a profit is earned, and

profit = selling price – cost.

Similarly, if
the cost > the selling price, then a loss is incurred and
loss = cost – selling price.

Example 1

A company which produces shoes incurs a cost of Rs 1000 in manufacturing a pair of shoes. The company sells each pair of shoes at Rs 2600. Find the profit earned by the company in selling one pair of shoes.

The manufacturing cost of a pair of shoes = Rs 1 000

Selling price = Rs 2 600

$\therefore$ Profit earned = Rs 2 600 − 1 000

= Rs $\underline{\underline{1\,600}}$

Example 2

A vendor buys a stock of fifty coconuts at the price of Rs 45 per coconut. If the vendor sells all the coconuts at the price of Rs 60 per fruit, calculate his profit.

Method I

The buying price of the stock of coconuts = Rs 45 × 50

= Rs 2 250

Income generated by selling the stock of coconuts = Rs 60 × 50

= Rs 3 000

$\therefore$ The profit earned by selling the stock of coconuts = Rs 3 000 – 2 250

= $\underline{\underline{\text{Rs } 750}}$

Method II

The purchase price of a coconut = Rs 45

The selling price of a coconut = Rs 60

The profit earned by selling one coconut = Rs 60 – 45

= Rs 15

The profit earned by selling the whole stock of coconuts = Rs 15 × 50

= $\underline{\underline{\text{Rs } 750}}$

Example 3

A vendor buys a stock of 100 mangoes at the price of Rs 20 each and decides to sell them at the price of Rs 18 each due to the fruits being damaged during transportation. Calculate the loss incurred by the vendor.

Method I

The purchase price of the stock of mangoes $=$ Rs 20×100
$=$ Rs 2 000

The amount made by selling the stock of mangoes $=$ Rs 18×100
$=$ Rs 1 800

The loss incurred in selling the whole stock of mangoes $=$ Rs 2 000 – 1 800
$=$ Rs 200

Method II

The purchase price of a mango $=$ Rs 20

The selling price of a mango $=$ Rs 18

The loss incurred in selling a mango $=$ Rs 20 – 18
$=$ Rs 2

The loss incurred in selling the whole stock of mangoes $=$ Rs 2×100
$=$ Rs 200

Example 4

A vendor buys 60 kg of manioc from a farmer at the price of Rs 50 per kilogramme. He initially sells 20 kg at Rs 70 per kilogramme. Of the remaining manioc he sells 15 kg at Rs 60 per kilogramme, 5 kg at Rs 50 per kilogramme and finally 10 kg at Rs 40 per kilogramme. The vendor discards the remaining 10 kg of manioc due to his inability to sell it. Determine whether the vendor earned a profit or incurred a loss from selling the manioc and calculate the profit earned or loss incurred by him.

The cost incurred in buying the manioc $=$ Rs 50×60
$=$ Rs 3 000

The amount made by selling the first 20 kg of manioc $=$ Rs 70×20
$=$ Rs 1 400

The amount made by selling the next 15 kg of manioc $= \text{Rs } 60 \times 15$
$= \text{Rs } 900$

The amount made by selling 5 kg of manioc $= \text{Rs } 50 \times 5$
$= \text{Rs } 250$

The amount made by selling 10 kg of manioc $= \text{Rs } 40 \times 10$
$= \text{Rs } 400$

The amount made by selling the whole stock of manioc $= \text{Rs } 1400 + 900 + 250 + 400$
$= \text{Rs } 2950$

Since $3000 > 2950$, a loss is incurred by the vendor.

The loss incurred by the vendor $= \text{Rs } 3000 - 2950$
$= \underline{\underline{\text{Rs } 50}}$

Exercise 4.1

1. Fill in the blanks based on the given information.

Item	Purchase price/ Production cost (Rs)	Selling price (Rs)	Whether it is a profit or a loss	Profit/Loss (Rs)
Wristwatch	500	750	....	....
School Bag	1 200	1 050	....	....
Calculator	....	1 800	Profit	300
Drink Bottle	....	750	Loss	175
Water Bottle	350	....	Loss	50
Box of mathematical instruments	275	....	Profit	75
Umbrella	....	450	Loss	100
Pair of Slippers		700	Profit	150

2. Find the more profitable business of each pair given below.

i. Selling mangoes at Rs 60 per fruit which were bought at Rs 50 per fruit.
Selling oranges at Rs 55 per fruit which were bought at Rs 50 per fruit.

ii. Selling coconuts at Rs 60 per fruit which were bought at Rs 40 per fruit.
Selling jack fruits at Rs 60 per fruit which were bought at Rs 50 per fruit.

iii. Selling pens at Rs 15 each which were bought at Rs 10 each.
Selling books at Rs 28 each which were bought at Rs 25 each.

3. A vendor buys a stock of 100 rambutans at the price of Rs 3 per fruit. He discards 10 fruits which are spoilt and sells the remaining stock at the price of Rs 5 per fruit. Determine whether the vendor earns a profit or incurs a loss and calculate the profit earned or loss incurred by him.

4. A vendor buys a stock of 50 kg of beans at the price of Rs 60 per kilogramme. On the first day he sells 22 kg of beans at the price of Rs 75 per kilogramme and on the second day he sells the remaining stock at the price of Rs 70 per kilogramme.

i. Calculate the profit earned by the vendor on each day and determine on which day he earned a greater profit.

ii. Calculate his total profit.

5. The production cost of a cane chair is Rs 650. A manufacturer produces 20 such chairs. He expects to earn a profit of Rs 7 000 by selling all the chairs. In order to do this, what should be the selling price of a chair?

6. A vendor, who sells apples by the roadside after buying them from a wholesaler, buys 200 apples on a certain day at the price of Rs 25 per fruit. He expects to earn a profit of Rs 1000 by selling the whole stock of apples. In order to do this, determine the price at which he should sell a fruit.

7. A vendor bought a stock of 50 kg of onions at the price of Rs 60 per kilogramme and sold 30 kg of it at the price of Rs 80 per kilogramme. He had to sell the remaining stock of onions at a lesser price because they were close to getting spoilt. Due to this, the vendor neither made a profit nor incurred a loss from selling the whole stock of onions. Find the price at which the vendor sold each kilogramme of the remaining stock of onions.

4.2 Profit percentage/loss percentage

Ramesh and Suresh are two vendors. Ramesh owns a clothing store. He sells a pair of trousers which he bought for Rs 800, at the price of Rs 900. Suresh owns an electrical items store. He sells an electric kettle which he bought for Rs 2500, at the price of Rs 2600.

The items sold by Ramesh and Suresh are not the same, and the buying prices and selling prices of the items are also different. However, the profit earned by them from selling the items is equal.

The profit earned by Ramesh from selling a pair of trousers = Rs 900 – 800
= Rs 100

The profit earned by Suresh from selling an electric kettle = Rs 2600 – 2500
= Rs 100

Can you identify which of these two sellers engaged in the more profitable sale if both had Rs 5000 each?

Even though the profit earned by Ramesh and Suresh is equal, it is clear that the amount of money each person spent in order to earn that profit is not equal. In order to find out which was the more profitable sale, the amount of money spent by each person has to be considered. The below given calculation is performed in order to determine this.

The profit earned by Ramesh after spending Rs 800 = Rs 100

The profit earned by Ramesh as a fraction of the amount he spent $= \frac{100}{800}$

The profit earned by Suresh after spending Rs 2500 = Rs 100

The profit earned by Suresh as a fraction of the amount he spent $= \frac{100}{2500}$

It is easy to compare the fractions $\frac{100}{800}$ and $\frac{100}{2500}$ since the numerators of both fractions are equal. Since $\frac{100}{800} > \frac{100}{2500}$ Ramesh's transaction was more profitable.

Even when the numerators are not equal, the more profitable business is determined using a similar method. Since the comparison of fractions when the denominators are different could be difficult, these fractions are most often converted into percentages to facilitate comparison. Let us calculate these percentages as follows.

Since the profit earned by Ramesh written as a fraction of the cost is $\frac{100}{800}$,

$$\text{Ramesh's profit percentage} = \frac{100}{800} \times 100\%$$
$$= \underline{\underline{12.5\%}}.$$

Accordingly, it is clear that the profit earned by Ramesh from spending Rs 100 is Rs 12.50.

Since the profit earned by Suresh written as a fraction of the cost is $\frac{100}{2500}$,

$$\text{Suresh's profit percentage} = \frac{100}{2500} \times 100\%$$
$$= \underline{\underline{4\%}}.$$

Accordingly, it is clear that the profit earned by Suresh from spending Rs 100 is Rs 400.

Since 12.5% > 4%, it can be said that Ramesh's transaction was more profitable.

The meaning of the percentages calculated above can be described as follows.

$\frac{100}{800} \times 100$ is the profit Ramesh earns from spending Rs 100.

$\frac{100}{2500} \times 100$ is the profit Suresh earns from spending Rs 100.

The profit earned/loss incurred by a vendor when the buying price/production cost of the item is Rs 100, is known as the profit/loss percentage. Therefore, by representing the profit/loss as a fraction of the buying price/production cost and multiplying that fraction by 100%, the profit/ loss percentage can be calculated.

$$\text{Profit percentage} = \frac{\text{profit}}{\text{buying price (or production cost)}} \times 100\%$$

$$\text{Loss percentage} = \frac{\text{loss}}{\text{buying price (or production cost)}} \times 100\%$$

Example 1

A vendor buys exercise books at Rs 25 each, and sells them at Rs 30 each. Calculate the profit percentage earned by the vendor from selling one exercise book.

$$\text{Profit} = \text{Rs } 30 - 25$$
$$= \text{Rs } 5$$

$$\text{Profit percentage} = \frac{5}{25} \times 100\%$$
$$= 20\%$$

Example 2

A vendor buys a pair of trousers for Rs 500. Due to a damage, he sells it for Rs 450. Determine the loss percentage.

$$\text{Loss} = \text{Rs } 500 - 450$$
$$= \text{Rs } 50$$

$$\text{Loss percentage} = \frac{50}{500} \times 100\%$$
$$= 10\%$$

Example 3

A carpenter incurs a cost of Rs 4000 in making a table which he sells at Rs 5600. A blacksmith incurs a cost of Rs 250 in making a knife which he sells at Rs 360. Determine who has engaged in the more profitable sale.

The profit earned by the carpenter as a percentage of the cost incurred $= \frac{1600}{4000} \times 100\% = 40\%$

The profit earned by the blacksmith as a percentage of the cost incurred $= \frac{110}{250} \times 100\% = 44\%$

Therefore, the blacksmith's transaction was more profitable.

Example 4

If a vendor buys an almirah for Rs 30 000 and earns a profit percentage of 15% (of the purchase price) by selling it, calculate the selling price of the almirah.

Method I

Here, what is meant by a profit percentage of 15% is that, if Rs 100 is invested, then a profit of Rs 15 is earned. In other words, if Rs 100 is invested, then the item is sold at the price of Rs 115.

Therefore, the selling price of the item when Rs 30 000 is invested $= \frac{115}{100} \times 30\,000$

$= \underline{\underline{\text{Rs } 34\,500}}$

Method II

As in method I above,
since the profit is Rs 15 when Rs 100 is invested,

the profit earned when Rs 30 000 is invested $= \frac{15}{100} \times 30\,000$

$= \text{Rs } 4\,500$

Therefore, the selling price of the item = cost + profit

$= 30\,000 + 4\,500$

$= \underline{\underline{\text{Rs } 34\,500}}$

Example 5

A vendor buys a pair of shoes for Rs 1500 and sells it at a loss of 2%. What is the selling price of the pair of shoes?

Method I

Since the pair of shoes is sold at a loss of 2%,

the selling price if the item is worth Rs 100 = Rs 98

$\therefore$ The selling price of the item worth Rs 1 500 $= \text{Rs } \frac{98}{100} \times 1\,500$

$= \underline{\underline{\text{Rs } 1\,470}}$

Method II

$$\text{The loss incurred} = \text{Rs } 1\,500 \times \frac{2}{100}$$
$$= \text{Rs } 30$$
$$\therefore \text{ The selling price} = \text{Rs } 1\,500 - 30$$
$$= \underline{\underline{\text{Rs } 1\,470}}$$

Example 6

If a vendor earns a profit of 10% by selling a television set at the price of Rs 22 000, find the price at which the vendor bought the set.

Method I

In order to earn a profit of 10% when the purchase price of the item is Rs 100, the item should be sold for Rs 110.

Therefore, the purchase price of an item sold for Rs 110 at a profit of 10% = Rs 100

$\therefore$ The purchase price of an item sold for Rs 22 000 at a profit of 10% $= \text{Rs } \frac{100}{110} \times 22\,000$

$$= \underline{\underline{\text{Rs } 20\,000}}$$

Method II

If the purchase price of the item is Rs x, then

$$\text{the profit earned} = \text{Rs } x \times \frac{10}{100}$$
$$= \text{Rs } \frac{x}{10}$$

$$\text{The selling price of the item} = \text{Rs } x + \frac{x}{10}$$
$$\therefore x + \frac{x}{10} = 22\,000$$
$$\frac{10x + x}{10} = 22\,000$$
$$\frac{11x}{10} = 22\,000$$
$$x = 22\,000 \times \frac{10}{11}$$
$$x = 20\,000$$

Therefore, the purchase price of the television set is Rs 20 000.

Method III

If the purchase price is Rs x,

$$\text{the selling price} = \text{Rs } x \times \frac{110}{100}$$

$$\therefore\ x \times \frac{110}{100} = 22\ 000$$

$$x = 20\ 000$$

Therefore, the purchase price of the set is Rs 20 000.

Example 7

A vendor had to sell a sports item for Rs 6 800 due to a manufacturing defect, which caused him a loss of 15%. Find the purchase price of the item.

Method I

The selling price of an item which is bought at Rs 100 and sold at a loss of 15%, is Rs 85.

$\therefore$ The purchase price of an item sold at Rs 85 at a loss of 15% = Rs 100

Hence, the purchase price of an item sold at Rs 6 800 at a loss of 15% $= \text{Rs } \frac{100}{85} \times 6\ 800$

$$= \underline{\underline{\text{Rs } 8\ 000}}$$

Method II

If the purchase price of the item is Rs x,

$$\text{the loss incurred} = \text{Rs } x \times \frac{15}{100}$$

$$= \text{Rs } \frac{3x}{20}$$

$\therefore$ The selling price of the item

$$= \text{Rs } x - \frac{3x}{20}$$

$$\text{Then, } x - \frac{3x}{20} = 6\ 800$$

$$\frac{20x - 3x}{20} = 6\ 800$$

$$\frac{17x}{20} = 6\ 800$$

$$x = 6\ 800 \times \frac{20}{17}$$

$$\underline{\underline{x = 8\ 000}}$$

$\therefore$ The purchase price of the item is Rs 8000.

Exercise 4.2

1. Fill in the blanks in the table based on the information that is given.

	Purchase price (Rs)	Selling price (Rs)	Whether it is a profit or a loss	Profit/ Loss (Rs)	Profit/ Loss percentage
i.	400	440	Profit	40	10%
ii.	600	720	....	....	....
iii.	1500	1200	....	....	....
iv.	60	....	Profit	....	60%
v.	180	....	Profit	....	30%
vi.	150	75	Loss	....	....
vii.	200	....	Loss	....	10%

2. If a vendor buys a pair of trousers for Rs 500 and sells it at Rs 650, determine

i. his profit,

ii. the profit percentage.

3. If an electric iron which is worth Rs 2500 is sold at the price of Rs 2300, determine

i. the loss,

ii. the loss percentage.

4. A vendor buys a stock of 100 mangoes at the price of Rs 18 each. He discards 20 mangoes due to them being spoilt and sells the rest of the stock at the price of Rs 30 per fruit. Determine whether he has earned a profit or incurred a loss and calculate,

i. the profit earned/ loss incurred,

ii. profit/ loss percentage.

5. The production costs of several types of clothing produced and sold by a certain tailor, together with their selling prices are given in the table below.

The types of clothing	**Production cost (Rs) per item**	**Selling price (Rs) per item**
Shirts	300	350
Pairs of trousers	400	450
Frocks	500	575
Raincoats	1000	1150

i. For each of the above items, find the profit and the profit percentage earned by the tailor.

ii. Giving reasons, write the most profitable item that is produced by the tailor.

6. If a bookseller earns a profit of 25% by selling a novel worth Rs 300, calculate the selling price of the novel.

7. If a bicycle worth Rs 12 000 is sold at a loss of 10%, calculate the selling price of the bicycle.

8. A carpenter spends Rs 1800 in producing a chair. He sells the chair to a vendor at a profit of 20%. The vendor then sells the chair to a customer at a profit of 20%.

i. How much does the vendor spend to buy the chair?

ii. How much does the customer spend to buy the chair?

iii. Write with reasons whether the carpenter or the vendor earns a greater profit.

9. If a vendor earns a profit of 10% by selling a refrigerator for Rs 33 000, calculate its purchase price.

10. If a vendor incurs a loss of 5% by selling an electric stove for Rs 28 500, calculate its purchase price.

11. The profit/loss percentages of several items sold by a vendor and their selling prices are given in the table below. Calculate the purchase price of each item.

Item	**Selling price**	**Profit percentage**	**Loss percentage**
Clock	3 240	8%	-
Electric stove	7 500	25%	-
Camera	12 048	-	4%

4.3 Discounts and Commissions

Discounts

A discount of 20% is given on
every book

The price at which an item is expected to be sold is called the marked price. According to the Consumer Affairs Authority Act, the price of an item that is for sale needs to be marked on the item.

A notice displayed in a bookshop is given in the picture shown above. What is mentioned in the notice is that a discount of 20% is given when a book is bought. This means that, when the book is purchased, 20% will be reduced from the price mentioned on the book. The amount that is reduced is called a "**discount**". Most often, a discount is indicated as a percentage of the marked price.

Since customers usually tend to buy goods from shops which offer discounts, there is an increase in sales in these shops. Due to this, the profits of the shop also increase. Discounts result in direct benefits for customers while the shop owners too gain long term benefits.

Example 1

Kaveesha buys books which are worth Rs 1500 from a bookshop which offers a discount of 20%. Calculate the discount that Kaveesha receives.

$$\text{Discount} = \text{Rs } 1\,500 \times \frac{20}{100}$$
$$= \text{Rs } 300$$

Example 2

The production cost of a mobile phone is Rs 9000. The price of the phone has been marked keeping a profit of Rs 3000. If the phone is sold at a discount of 10% on the marked price, find the selling price.

Method I

The marked price = Rs 9000 + 3000
= Rs 12 000

$$\text{Discount} = \text{Rs } 12\,000 \times \frac{10}{100}$$
$$= \text{Rs } 1\,200$$
$$\therefore \text{ Selling price} = \text{Rs } 12\,000 - 1\,200$$
$$= \underline{\underline{\text{Rs } 10\,800}}$$

Method II

Since an item of marked price Rs 100 is sold for Rs 90 when the discount is 10%,

the selling price of an item of marked price Rs 100, sold at a discount of 10% = Rs 90

$\therefore$ The selling price of an item of marked price Rs 12 000, sold at a discount of 10% $= \text{Rs } \frac{90}{100} \times 12\,000$

$$= \underline{\underline{\text{Rs } 10\,800}}$$

Note: In the above example, two methods of solving the problem have been given. Since the second method presented is shorter than the first, it is important to practice this method.

Example 3

A discount of Rs 250 is given on the marked price of Rs 2000 when a certain wristwatch is bought. Find the discount percentage offered.

$$\text{The discount percentage} = \frac{250}{2000} \times 100\%$$
$$= \underline{\underline{12.5\%}}$$

Example 4

If a storybook is sold for Rs 460 at a discount of 8%, what is the marked price?

$$\text{The marked price} = \text{Rs } 460 \times \frac{100}{92}$$
$$= \underline{\underline{\text{Rs } 500}}$$

Commissions

A notice issued by a company which facilitates the sale of properties, vehicles and houses is shown in the above picture. While such companies find customers for these kinds of sales, once the deal is over, a certain percentage of the value of the transaction is charged by the company. Such companies are known as brokerages. The amount that is charged by such companies for facilitating the sale is known as the commission. This is usually a percentage of the value of the transaction.

Example 5

What is the commission charged by a company for facilitating the sale of a motorcar worth Rs 3 000 000, if a commission of 5% is charged?

$$\text{The commission charged} = \text{Rs } 3\,000\,000 \times \frac{5}{100}$$
$$= \underline{\underline{\text{Rs } 150\,000}}$$

Example 6

A real estate company charges a fee of Rs 36,000 to facilitate the sale of a land worth Rs 1 200 000. Calculate the commission percentage charged by the company.

$$\text{The commission percentage} = \frac{36\,000}{1\,200\,000} \times 100\%$$
$$= \underline{\underline{3\%}}$$

Exercise 4.3

1. A discount of 5% is offered when a television set of marked price Rs 25 000 is purchased.
   (i) How much is offered as the discount (in rupees) ?
   (ii) Find the selling price of the television set.

2. Nimithee buys a pair of trousers worth Rs 1 500 and a shirt worth Rs 1 200 from a shop which offers a discount of 5%. How much does Nimithee have to pay for both the items?

3. Two notices displayed during the festive season in two shoe shops which sell the same types of shoes are given below.

Shop A	**Shop B**
A discount of 8% on all purchases	**A reduction of Rs 100 on all purchases of value greater than Rs 1000**

   i. How much needs to be paid when purchasing a pair of shoes of marked price Rs 1 500 from shop A?

   ii. How much needs to be paid when purchasing a pair of shoes of marked price Rs 1 500 from shop B?

   iii. What is the discount percentage offered by shop B for this pair of shoes?
   iv. Is it more beneficial for the customer to buy the pair of shoes from shop A or from shop B?

4. A seller of bicycles buys a bicycle for Rs 8 000 and marks its selling price so that he earns a profit of 25%. When selling the bicycle, if the payment is done outright, a discount of 10% is offered to the customer.
   i. Find the marked price of the bicycle.
   ii. Find the price of the bicycle when the discount is given.
   iii. If the seller marks the selling price so that he earns a profit of 20% on the amount he paid for the bicycle, then find its selling price.

5. A vendor marks the price of an item such that he earns a profit of 10%. He intends to offer a discount of 10% on the marked price when the item is sold. Describe the profit earned or loss incurred by the vendor at the sale of the item.

6. A company charges a commission of 3% on the sale of a land. When selling a land worth Rs 5 000 000,

   i. how much needs to be paid as the commission?
   ii. how much does the land owner receive after paying the commission?

7. If a broker charged Rs 25 000 for selling a generator which was worth Rs 300 000, calculate the commission percentage that he charged.

8. A person who sells his vehicle is left with Rs 570 000 after paying Rs 30 000 to the broker.

   i. What is the selling price of the vehicle?
   ii. What is the commission percentage charged by the broker?

9. A person paid a commission of 3% when he purchased a house. If he paid Rs 54 000 as commission, find the amount he paid for the house.

Miscellaneous Exercise

1. Kasun decides to sell 10 perches of a land he owns at the price of Rs 300 000 per perch. He promises a commission of 3% on the sale of the land to a broker. If he gives a discount of 1% on the original price to the buyer, find his income from the sale of the land, after paying the commission to the broker.

2. Amal who is a car dealer purchases a car for Rs 5 000 000. He intends selling the car for Rs 6 000 000. However, he gives a discount of 3% on this price to the buyer and a commission of 2% to a broker. Determine Amal's profit.

Summary

Summary

- profit = selling price – cost
- loss = cost – selling price
- Profit percentage $= \dfrac{\text{profit}}{\text{buying price (or production cost)}} \times 100\%$
- Loss percentage $= \dfrac{\text{loss}}{\text{buying price (or production cost)}} \times 100\%$

5 Algebraic Expressions

By studying this lesson, you will be able to;

- find the value of simple algebraic expressions by substituting directed numbers,
- expand the product of two binomial expressions of the form $(x \pm a)\,(x \pm b)$,
- verify the expansion of the product of two binomial expressions by considering areas.

Algebraic expressions

Do the following exercise to review what you have learnt in grade 8, related to algebraic expressions.

Review Exercise

1. Expand the following expressions.

a. $5\,(x + 2)$ **b.** $3\,(y + 1)$ **c.** $4\,(2m + 3)$
d. $3\,(x - 1)$ **e.** $4\,(3 - y)$ **f.** $2\,(3x - 2y)$
g. $-2\,(y + 3)$ **h.** $-3\,(2 + x)$ **i.** $-5\,(2a + 3b)$
j. $-4\,(m - 2)$ **k.** $-(5 - y)$ **l.** $-10\,(-3b - 2c)$

2. Expand the following expressions.

a. $x\,(a + 2)$ **b.** $y\,(2b - 3)$ **c.** $a\,(2x + 3y)$
d. $2a\,(x + 5)$ **e.** $2b(y - 2)$ **f.** $3p\,(2x - y)$
g. $(-3q)\,(p + 8)$ **h.** $(-2x)\,(3 - 2y)$ **i.** $(-5m)\,(x - 2y)$

3. Find the value of each of the following expressions when $x = 3$ and $y = -2$.

a. $x + y$ **b.** $x - y$ **c.** $3x - 2y$
d. $-2x + y$ **e.** $2(x + y)$ **f.** $3\,(2x - y)$

4. Expand and simplify each of the following expressions.

a. $3\,(x + y) + 2\,(x - y)$ **b.** $5(a + b) + 4\,(a + c)$
c. $4\,(a + b) + 3\,(2a - b)$ **d.** $2(a - b) + (2a - b)$
e. $5\,(m + n) + 2\,(m + n)$ **f.** $3(m + n) - (m - n)$
g. $5\,(x - y) - 3\,(2x + y)$ **h.** $2(3p - q) - 3\,(p - q)$
i. $-4\,(m + n) + 2\,(m + 2)$ **j.** $-4\,(a - b) - 2\,(a - b)$

5.1 Substitution

In grade 8, you learnt to find the value of an algebraic expression by substituting integers for the unknown terms. Let us now find out how to obtain the value of an algebraic expression by substituting directed numbers.

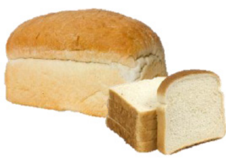
- 20 adults and 16 children went on a trip. Each adult was given x amount of bread and each child was given y amount of bread for breakfast.

Let us write the total amount of bread that was distributed as an algebraic expression.

Amount of bread given to the 20 adults $= 20x$
Amount of bread given to the 16 children $= 16y$
Total amount of bread that was distributed $= 20x + 16y$

Let us find out the total amount of bread that was distributed, if an adult was given half a loaf of bread and a child was given a quarter loaf of bread.

Then $x = \frac{1}{2}$ and $y = \frac{1}{4}$. To find out the total amount of bread that was distributed, $x = \frac{1}{2}$ and $y = \frac{1}{4}$ should be substituted in the expression $20x + 16y$.



Accordingly, the total number of loaves of bread that were distributed $= 20 \times \frac{1}{2} + 16 \times \frac{1}{4}$
$= 10 + 4$
$= 14$

Example 1

Find the value of each of the following algebraic expressions when $a = \frac{1}{2}$.

i. $2a + 3$

$$\begin{aligned} 2a + 3 &= 2 \times \frac{1}{2} + 3 \\ &= 1 + 3 \\ &= \underline{\underline{4}} \end{aligned}$$

ii. $6 - 4a$

$$\begin{aligned} 6 - 4a &= 6 - 4 \times \frac{1}{2} \\ &= 6 - 2 \\ &= \underline{\underline{4}} \end{aligned}$$

iii. $3a - 1$

$$\begin{aligned} 3a - 1 &= 3 \times \frac{1}{2} - 1 \\ &= \frac{3}{2} - 1 \\ &= \frac{3 - 2}{2} \\ &= \underline{\underline{\frac{1}{2}}} \end{aligned}$$

Example 2

Find the value of each of the following algebraic expressions when $b = -\frac{2}{3}$.

i. $3b + 5$

$3b + 5$

$= 3 \times \frac{-2}{3} + 5$

$= (-2) + 5$

$= \underline{\underline{3}}$

ii. $5 - 6b$

$5 - 6b$

$= 5 - 6 \times (-\frac{2}{3})$

$= 5 + (-6) \times (-\frac{2}{3})$

$= 5 + 4$

$= \underline{\underline{9}}$

iii. $2b + \frac{1}{3}$

$2b + \frac{1}{3}$

$= 2 \times (\frac{-2}{3}) + \frac{1}{3}$

$= \frac{-4}{3} + \frac{1}{3}$

$= \frac{-3}{3}$

$= \underline{\underline{-1}}$

Example 3

Find the value of each of the following algebraic expressions when $x = \frac{1}{2}$ and $y = -\frac{1}{4}$.

i. $2x + 4y$

$2x + 4y = 2 \times \frac{1}{2} + 4 \times \left(\frac{-1}{4}\right)$

$= 1 - 1$

$= \underline{\underline{0}}$

ii. $2x - 2y$

$2x - 2y = 2 \times \frac{1}{2} - 2 \times \left(\frac{-1}{4}\right)$

$= 1 + \frac{1}{2}$

$= \underline{\underline{1\frac{1}{2}}}$

iii. $4xy$

$4xy = 4 \times \frac{1}{2} \times \left(\frac{-1}{4}\right)$

$= \underline{\underline{\frac{-1}{2}}}$

iv. $-2xy$

$-2xy = -2 \times \left(\frac{1}{2}\right) \times \left(\frac{-1}{4}\right)$

$= \underline{\underline{\frac{1}{4}}}$

Exercise 5.1

1. Find the value of each of the following algebraic expressions when $x = \frac{1}{4}$.

i. $4x$ **ii.** $2x$ **iii.** $3x$ **iv.** $-8x$

2. Find the value of each of the following algebraic expressions when $y = \frac{-1}{3}$.

i. $3y$ **ii.** $2y$ **iii.** $-6y$ **iv.** $-4y$

3. Find the value of each of the following algebraic expressions when $a = -2$ and $b = \frac{1}{2}$.

i. $a + 2b$ **ii.** $4b - a$ **iii.** $3a + b$

4. Find the value of each of the following algebraic expressions when $x = \frac{2}{3}$ and $y = \frac{3}{4}$.

i. $3x + 4y$ **ii.** $3x - 2y$ **iii.** $8y - 6x$

5. Find the value of each of the following algebraic expressions when $p = -\frac{1}{2}$ and $q = -3$.

i. $2p + q$ **ii.** $4p - q$ **iii.** $6pq - 2$

5.2 The product of two binomial expressions

Let us first recall what is meant by algebraic symbols, algebraic terms, algebraic expressions and binomial expressions. The letters $x, y, z, a, b, c, \ldots$are considered as algebraic symbols.

Algebraic symbols such as x, y and z are also considered as algebraic terms.
When an algebraic symbol is multiplied or divided by a number, as for example, $2x$, $5y$, $-2a$ and $\frac{x}{3}$, it too is considered as an algebraic term.
Similarly, when an algebraic symbol is multiplied or divided by another algebraic symbol, as for example, xy, ay and $\frac{b}{z}$, it is also called an **algebraic term**. The products and quotients of algebraic symbols and numbers such as $2xy$, $-3zab$ and $\frac{2}{5}xy$ are also called **algebraic terms**.

Algebraic terms can also be considered as algebraic expressions (expressions with one term).

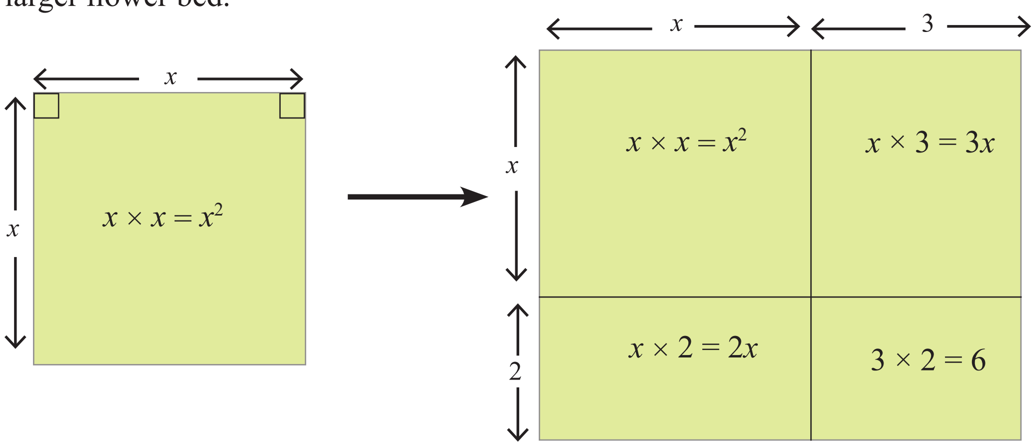
A sum or a difference of algebraic terms is called an **algebraic expression**. For example, $x + y$, $2a + xyz$, $4xy^2 - yz$ and $-2x + 3xy$ are algebraic expressions. Similarly, when a number is added to or subtracted from an algebraic term, it is also called an algebraic expression. For example, $4 + x$ and $1 - 3ab$ are **algebraic expressions**.

All the algebraic expressions we have considered thus far have consisted of two terms. A "binomial algebraic expression" (or simply a "binomial expression) is an expression which is a sum or difference of two terms.

However, there can be any number of terms in an algebraic expression.
$3 + ax - 2xyz + xy$ is an algebraic expression with four terms. It has three algebraic terms and a number (constant term).

In this lesson we will be studying binomial expressions. Now let us consider the product of two binomial expressions.

Let us take the length of a side of the square shaped flower bed shown in the figure below as x units. If a larger rectangular flower bed is made by increasing the length of one side by 3 units and the length of the adjacent side by 2 units, let us consider how an algebraic expression can be constructed in terms of x, for the area of the larger flower bed.



The length of the larger flower bed $= x + 3$ units
The breadth of the larger flower bed $= x + 2$ units

According to the figure,
the area of the larger flower bed = length × breadth= $(x + 3)(x + 2)$ square units ——(1)

Observe that $(x + 3)(x + 2)$ is a product of two binomial expressions.

The area of the larger flower bed can also be found by using a different method, that is, by adding the areas of the four smaller sections of which it is composed. The four sections are, the initial square shaped section and the three smaller rectangular sections in the figure.

Accordingly,

the area of the larger flower bed = the sum of the areas of the four smaller sections

$= x^2 + 2x + 3x + 6$ square units

$= x^2 + 5x + 6$ square units ———— (2)

Irrespective of the method used to find the area, the expressions obtained for the area should be equal to each other. Therefore, from (1) and (2) the following equality is established.

$$(x + 3)(x + 2) = x^2 + 5x + 6$$

Let us now consider how this equality can be obtained without the aid of a figure.

Let us multiply the terms within the second pair of brackets by the two terms within the first pair of brackets.

$$(x + 3)(x + 2) = (x + 3)(x + 2)$$

$$= x(x + 2) + 3(x + 2)$$

$$= x^2 + 2x + 3x + 6$$

$$= x^2 + 5x + 6$$

Accordingly, the product of two binomial expressions can be obtained in the above manner without the aid of a figure.

Let us consider another activity similar to the one above.

Activity 1

Fill in the blanks using the given information.

A square shaped metal sheet of side length x centimetres is shown in Figure I. Figure II illustrates how two strips of width 2 centimetres and 3 centimetres respectively have been cut off from the two sides of the sheet.

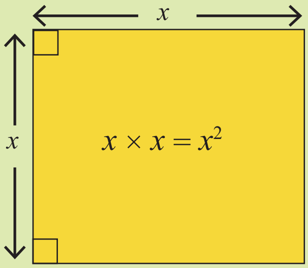


Figure I

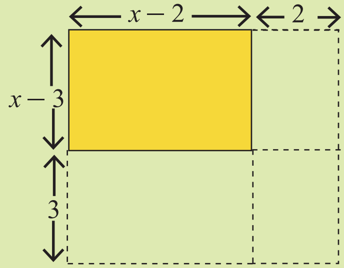


Figure II

The area of the remaining rectangular sheet $= (x-2)\,(x-3)$

According to Figure II,

the area of the remaining rectangular sheet = the area of the square − the area of the three rectangular parts—②

$$= x^2 - 2\,(.........) - ...\,(x-2) - 2 \times 3$$

Accordingly, $(x-2)\,(x-3) = x^2 - 2(.........) - ...(x-2) - 2 \times 3$

$= ....$

$= ....$

Let us consider a few examples to develop a better understanding of how the product of two binomial expressions is obtained.

Example 1

$(x+5)\,(x+3)$

$$\begin{aligned}(x+5)\,(x+3) &= x\,(x+3) + 5\,(x+3)\\ &= x^2 + 3x + 5x + 15\\ &= \underline{\underline{x^2 + 8x + 15}}\end{aligned}$$

Example 2

$(x+5)\,(x-3)$

$$\begin{aligned}(x+5)\,(x-3) &= x\,(x-3) + 5\,(x-3)\\ &= x^2 - 3x + 5x - 15\\ &= \underline{\underline{x^2 + 2x - 15}}\end{aligned}$$

Example 3

$(x-5)\,(x+3)$

$$\begin{aligned}(x-5)\,(x+3) &= x\,(x+3) - 5\,(x+3)\\ &= x^2 + 3x - 5x - 15\\ &= \underline{\underline{x^2 - 2x - 15}}\end{aligned}$$

Example 4

$(x-5)\,(x-3)$

$$\begin{aligned}(x-5)\,(x-3) &= x\,(x-3) - 5\,(x-3)\\ &= x^2 - 3x - 5x + 15\\ &= \underline{\underline{x^2 - 8x + 15}}\end{aligned}$$

Example 5

Show that $(x+8)(x-3) = x^2 + 5x - 24$ when $x = 5$.

L.H.S. $= (x+8)(x-3)$

When $x = 5$

L.H.S. $= (5+8)(5-3)$

$= 13 \times 2$

$= 26$

R. H. S. $= x^2 + 5x - 24$

When $x = 5$

R. H. S. $= 25 + 25 - 24$

$= 26$

L.H.S. = R. H. S.

$\therefore (x+8)(x-3) = x^2 + 5x - 24$

Exercise 5.2

1. Expand and simplify each of the following products of binomial expressions.

a. $(x+2)(x+4)$ **b.** $(x+1)(x+3)$ **c.** $(a+3)(a+2)$
d. $(m+3)(m+5)$ **e.** $(p-4)(p-3)$ **f.** $(k-3)(k-3)$

2. Draw relevant rectangles for each product of binomial expressions in **a.**, **b.** and **e.** of **1.** above and verify the answers obtained in **1.** by calculating their areas.

3. Expand and simplify each of the following products of binomial expressions.

a. $(x+2)(x-5)$ **b.** $(x+3)(x-7)$ **c.** $(m+6)(m-1)$
d. $(x-2)(x+3)$ **e.** $(x-5)(x+5)$ **f.** $(m-1)(m+8)$
g. $(x-3)(x-4)$ **h.** $(y-2)(y-5)$ **i.** $(m-8)(m-2)$
j. $(x-3)(2-x)$ **k.** $(5-x)(x-4)$ **l.** $(2-x)(3-x)$

4. Join each of the expressions in column A, with the corresponding simplified expression in column B.

A	B
$(x+2)(x+1)$	$x^2 + 3x - 10$
$(x+3)(x-4)$	$x^2 - 25$
$(x+5)(x-2)$	$x^2 - 6x + 9$
$(x-3)(x-3)$	$x^2 + 3x + 2$
$(x-5)(x+5)$	$x^2 - x - 12$

5. Verify that $(x+5)(x+6) = x^2 + 11x + 30$ for each instance given below.

i. $x = 3$ **ii.** $x = -2$

6. Verify that $(x-2)(x+3) = x^2 + x - 6$, for each instance given below.

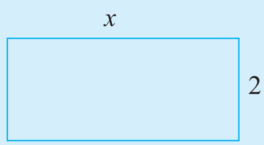
i. $x = 1$ **ii.** $x = 4$ **iii.** $x = 0$

7. Verify that $(2-x)(4-x) = x^2 - 6x + 8$, for each instance given below.

i. $x = 2$ **ii.** $x = 3$ **iii.** $x = -2$

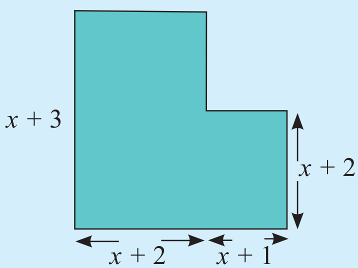
8. The length and breadth of a rectangular piece of decorative paper are 15 cm and 8 cm respectively. Two strips of breadth x cm each are cut off from the length and the breadth of this paper. Using a figure, obtain an expression for the area of the remaining portion. (Consider $x < 8$ cm).

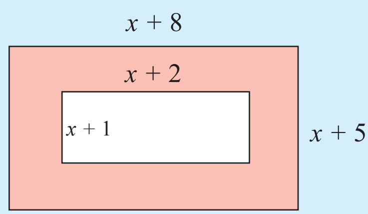
9. A rectangular flower bed of length x metres and breadth 2 metres is shown in the figure. Two metres are reduced from its length and x meters are added to its breadth. Construct an expression in terms of x for the area of the new flower bed by using a figure. (Consider that $x > 2$ m).



Miscellaneous Exercise

1. Write an expression for the shaded area in the given figure and simplify it.





2. If $(x+a)(x+4) = x^2 + bx + 12$, find the values of a and b.

6

Factors of Algebraic Expressions

By studying this lesson, you will be able to;
- factorize algebraic expressions with four terms when the factors are binomial expressions,
- factorize trinomial quadratic expressions of the form $x^2 + bx + c$,
- factorize algebraic expressions written as a difference of two squares.

Factors of algebraic expressions

The meanings of many algebraic terms were explained in the previous lesson. In this lesson we will consider what is meant by the factors of an algebraic expression (or an algebraic term).

Consider the term $2xy$. It is formed by the product of 2, x and y. Therefore, 2, x and y are all factors of $2xy$.

$2x + 2y$ is a binomial expression. It is the sum of two algebraic terms. 2 and x are factors of $2x$. Similarly, 2 and y are factors of $2y$. Accordingly, 2 is a factor of both the terms $2x$ and $2y$. You have learnt in grade 8 that the above binomial expression can be written as $2(x + y)$ by factoring out the common factor 2. Hence;

$$2x + 2y = 2(x + y)$$

What is important here is that the algebraic expression $2x + 2y$, which is the sum of $2x$ and $2y$, is expressed as a product of 2 and $x + y$. We say that 2 and $x + y$ are factors of $2x + 2y$. That is, the algebraic expression $2x + 2y$ can be expressed as a product of its factors 2 and $x + y$.

One factor of the above algebraic expression $2x + 2y$ is the number 2 and another factor is the algebraic expression $x + y$. However, an algebraic expression could also be expressed as a product of algebraic terms or algebraic expressions. For example, since the expression $xy + 5xz$ can be written as $x(y + 5z)$, x and $y + 5z$ are factors of it.

According to the facts learnt in lesson 5, when the algebraic expression $x(y + 5z)$ which is a product is expanded, we obtain the algebraic expression $xy + 5xz$, which is a sum of algebraic terms. In this lesson we will study the inverse of the process that was learnt in lesson 5. That is, we will learn how to write a given algebraic expression as a product of factors.

Observe how each algebraic expression given below has been written as a product of factors as learnt in grade 8.

- $3x + 12 = 3\,(x + 4)$
- $6a + 12b - 18 = 6\,(a + 2b - 3)$
- $-2x - 6y = -2\,(x + 3y)$
- $3x - 6xy = 3x\,(1 - 2y)$

In the second example above, the common factor of the terms of the expression $6a + 12b - 18$ is 6. Observe that this is the highest common factor of 6, 12 and 18. When a number is a common factor, we should always consider the highest common factor. Furthermore, when factorizing algebraic expressions, the numbers need not be factorized further. For example, $6x + 6y$ is written as $6(x + y)$ and not as $2 \times 3\,(x + y)$.

Do the following review exercise to establish these facts further.

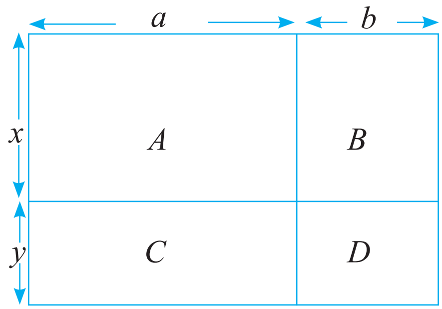
Review Exercise

Write each of the following algebraic expressions as a product of factors.

a. $8x + 12y$	**b.** $9a + 18y$	**c.** $3m + 6$
d. $20a - 30b$	**e.** $4p - 20q$	**f.** $12 - 4k$
g. $3a + 15b - 12$	**h.** $12a - 8b + 4$	**i.** $9 - 3b - 6c$
j. $-12x + 4y$	**k.** $-8a - 4b$	**l.** $-6 + 3m$
m. $ab + ac$	**n.** $p - pq$	**o.** $ab + ac - ad$
p. $3x + 6xy$	**q.** $6ab - 9bc$	**r.** $4ap + 4bp - 4p$
s. $x^3 + 2x$	**t.** $3m - 2nm^2$	**u.** $6s - 12s^2t$

6.1 Factors of algebraic expressions with four terms

The figure of a large rectangle which is composed of the four rectangular sections A, B, C and D is given below.



Let us find the area of each rectangle in terms of the given algebraic symbols x, y, a and b.

The area of section $A = a \times x = ax$

The area of section $B = b \times x = bx$

The area of section $C = a \times y = ay$

The area of section $D = b \times y = by$

Now let us find the area of the large rectangle.

The length of the large rectangle = $a + b$
The breadth of the large rectangle = $x + y$
Hence, the area of the large rectangle = $(a + b)(x + y)$

Now, since the total area of the 4 small rectangles = the area of the large rectangle, $ax + ay + bx + by = (a + b)(x + y)$.

We can verify the total above equality by expanding the product $(a + b)(x + y)$ by using the method learnt in the previous lesson.

Let us expand it as follows.

$$\begin{aligned}(a + b)(x + y) &= a(x + y) + b(x + y)\\ &= ax + ay + bx + by\end{aligned}$$

The validity of the equality is verified.

In this lesson we expect to learn how to write an expression of the form $ax + bx + ay + by$ as a product of two factors as $(a + b)(x + y)$. First, we need to observe that the four terms ax, ay, bx and by have no common factors. Therefore, the factoring out of a common factor cannot be done directly. However, if we consider the four terms pairwise, the expression can be factored as follows.

$$ax + bx + ay + by = (ax + bx) + (ay + by)$$
$$= x(a + b) + y(a + b)$$

The final expression is the sum of the two expressions $x(a + b)$ and $y(a + b)$. Now observe that the two expressions $x(a + b)$ and $y(a + b)$ have a common factor $(a + b)$. Therefore, by factoring out this expression, we can rewrite the given expression as a product of two factors as $(a + b)(x + y)$.

$$ax + bx + ay + by = x(a + b) + y(a + b)$$
$$= (a + b)(x + y)$$

Example 1

Factorize $3x + 6y + kx + 2ky$.

$$3x + 6y + kx + 2ky = 3(x + 2y) + k(x + 2y)$$
$$= \underline{\underline{(x + 2y)(3 + k)}}$$

Example 2

Factorize $a^2 - 3a + ab - 3b$.

$$a^2 - 3a + ab - 3b = a(a - 3) + b(a - 3)$$
$$= \underline{\underline{(a - 3)(a + b)}}$$

Example 3

Factorize $x^2 + xy - x - y$.

$$x^2 + xy - x - y = x^2 + xy - 1(x + y)$$
$$= x(x + y) - 1(x + y)$$
$$= \underline{\underline{(x + y)(x - 1)}}$$

Exercise 6.1

Factorize each of the following algebraic expressions.

a. $ax + ay + 3x + 3y$ **b.** $ax - 8a + 3x - 24$
c. $mp - mq - np + nq$ **d.** $ak + al - bk - bl$
e. $x^2 + 4x - 3x - 12$ **f.** $y^2 - 7y - 2y + 14$
g. $a^2 - 8a + 2a - 16$ **h.** $b^2 + 5b - 2b - 10$
i. $5 + 5x - y - xy$ **j.** $ax - a - x + 1$

6.2 Factors of trinomial quadratic expressions of the form of $x^2 + bx + c$

Recall how we obtained the product of the two algebraic expressions $(x + 3)$ and $(x + 4)$.

$$(x + 3)(x + 4) = x(x + 4) + 3(x + 4)$$
$$= x^2 + 4x + 3x + 12$$
$$= x^2 + 7x + 12$$

Since we have obtained $x^2 + 7x + 12$ as the product of $(x + 3)$ and $(x + 4)$, the expressions $(x + 3)$ and $(x + 4)$ are factors of $x^2 + 7x + 12$. Expressions of the form $x^2 + 7x + 12$ consisting of three terms of which one is a quadratic term are called trinomial quadratic expressions.

Note:
The trinomial quadratic expressions we consider here can in general be written in the form $x^2 + bx + c$. Here b and c are numerical values. For example, $x^2 + 7x + 12$ is the trinomial quadratic expression that is obtained when $b = 7$ and $c = 12$. In general, bx is called the middle term and c is called the constant term. The expression $x^2 + 7x + 12$ can be written as a product of two factors as $(x + 3)(x + 4)$. However, there are some trinomial quadratic expressions which cannot be written as a product of two such factors, as for example, the expression $x^2 + 3x + 4$.

Here, we only consider how to find the factors of those trinomial quadratic expressions that can be written as a product of two factors.

To find out how to write a trinomial quadratic expression as a product of two binomial terms, let us analyze the steps we carried out in obtaining the product of two binomial expressions, in the opposite direction.

- In the trinomial quadratic expression $x^2 + 7x + 12$, the middle term $7x$ has been written as a sum of two terms as $3x + 4x$.

  There are many ways of writing $7x$ as a sum of two terms. For example, $7x = 5x + 2x$ and $7x = 8x + (-x)$. The importance of $3x$ and $4x$ can be explained as follows.

- The product of $3x$ and $4x = 3x \times 4x = 12x^2$.
- Moreover, the product of the first and last terms of the trinomial quadratic expression $x^2 + 7x + 12$ is also $x^2 \times 12 = 12x^2$.

The observations from the above analysis can be used to factorize trinomial quadratic expressions. The middle term should be written as a sum of two terms. Their product should be equal to the product of the first and last terms of the expression to be factorized.

Let us factorize $x^2 + 6x + 8$. The middle term is $6x$. It should be written as a sum of two terms, and their product should be equal to $x^2 \times 8 = 8x^2$.

Based on the above facts, we have to find a pair of linear terms of which the product is $8x^2$ and the sum is $6x$. The table below shows the possible ways of writing $8x^2$ as a product of two linear terms.

Pair of linear terms	**Product**	**Sum**
$x, 8x$	$x \times 8x = 8x^2$	$x + 8x = 9x$
$2x, 4x$	$2x \times 4x = 8x^2$	$2x + 4x = 6x$

According to the table, it is clear that the middle term $6x$ is obtained from $2x + 4x$. Let us factorize the expression $x^2 + 6x + 8$.

$$\begin{aligned} x^2 + 6x + 8 &= x^2 + 2x + 4x + 8 \\ &= x(x + 2) + 4(x + 2) \\ &= (x + 2)(x + 4) \end{aligned}$$

$\therefore$ $x + 2$ and $x + 4$ are factors of $x^2 + 6x + 8$.

Instead of writing the middle term as $2x + 4x$, let us write it as $4x + 2x$ and factorize to see whether we obtain different factors.

$$\begin{aligned} x^2 + 6x + 8 &= x^2 + 4x + 2x + 8 \\ &= x(x + 4) + 2(x + 4) \\ &= (x + 4)(x + 2) \end{aligned}$$

The same pair of factors are obtained. Therefore, we see that the order in which the selected pair is written does not affect the final answer.

Example 1

Factorize $x^2 + 5x + 6$.
In this expression,
the product of the first and last terms $= x^2 \times 6 = 6x^2$
The middle term $= 5x$

We can factorize this expression as below, because $2x + 3x = 5x$ and $(2x)(3x) = 6x^2$

$$
\begin{aligned}
x^2 + 5x + 6 &= x^2 + 2x + 3x + 6 \\
&= x(x + 2) + 3(x + 2) \\
&= \underline{\underline{(x + 2)(x + 3)}}
\end{aligned}
$$

Example 2

Factorize $x^2 - 8x + 12$.

The product of the first and last terms of the expression is $x^2 \times 12 = 12x^2$ and the middle term is $(-8x)$. Here we have a negative term. The table given below shows the various ways in which two terms in x can be selected such that their product is $12x^2$.

x,	$12x$
$2x$,	$6x$
$3x$,	$4x$
$-2x$,	$-6x$
$-3x$,	$-4x$
$-x$,	$-12x$

According to the table, if we write $-8x = (-2x) + (-6x)$, then we obtain $(-2x)(-6x) = 12x^2$.

Hence,
$$
\begin{aligned}
x^2 - 8x + 12 &= x^2 - 2x - 6x + 12 \\
&= x(x - 2) - 6(x - 2) \\
&= \underline{\underline{(x - 2)(x - 6)}}
\end{aligned}
$$

Example 3

Factorize $y^2 + 2y - 15$.

The product of the first and last terms of the expression $= y^2 \times -15 = -15y^2$

The middle term $= 2y$

By writing $-15y^2 = (5y)(-3y)$, the middle term is obtained as $(5y) + (-3y) = 2y$

Therefore,

$$
\begin{aligned}
y^2 + 2y - 15 &= y^2 - 3y + 5y - 15 \\
&= y(y - 3) + 5(y - 3) \\
&= \underline{\underline{(y - 3)(y + 5)}}
\end{aligned}
$$

Example 4

Factorize $a^2 - a - 20$.

The product of the first and last terms of the expression is $= a^2 \times (-20) = -20a^2$ and the middle term is $(-a)$.

Since $-20a^2 = (-5a)(4a)$ and $(-5a) + (4a) = -a$, the expression can be factored as follows.

$$\begin{aligned} a^2 - a - 20 &= a^2 + 4a - 5a - 20 \\ &= a(a+4) - 5(a+4) \\ &= (a+4)(a-5) \end{aligned}$$

Exercise 6.2

Factorize the quadratic expressions given below.

a. $x^2 + 9x + 18$	**b.** $y^2 + 11y + 30$	**c.** $a^2 + 10a + 24$
d. $b^2 - 8b + 15$	**e.** $x^2 - 5x + 6$	**f.** $m^2 - 12m + 20$
g. $a^2 + a - 12$	**h.** $p^2 + 5p - 24$	**i.** $p^2 + 6p - 16$
j. $x^2 - x - 12$	**k.** $a^2 - 3a - 40$	**l.** $r^2 - 3r - 10$
m. $y^2 + 6y + 9$	**n.** $k^2 - 10k + 25$	**o.** $4 + 4x + x^2$
p. $36 + 15x + x^2$	**q.** $30 - 11a + a^2$	**r.** $54 - 15y + y^2$

Note:

When factorizing trinomial quadratic expressions, writing the middle term as a sum of two suitable terms is an important step. Although a specific method has been given above to find the two terms, an easier method is to write the middle term as a sum of two terms and check whether their product is equal to the product of the first and last terms of the given expression. This skill can be mastered with practice. However, once the two terms have been written, we have to be careful when simplifying the expression. In example 4 above, when the common factor -5 is factored out from the expression $-5a - 20$, we obtain $-5(a+4)$ This is often mistakenly written as $-5(a-4)$.

6.3 Factors of an expression written as a difference of two squares

Consider the product of the two binomial expressions $(x-y)$ and $(x+y)$.

$$\begin{aligned} (x-y)(x+y) &= x(x+y) - y(x+y) \\ &= x^2 + xy - xy - y^2 \\ &= x^2 - y^2 \end{aligned}$$

Accordingly, $(x+y)(x-y)$ is equal to the expression x^2-y^2. The expression x^2-y^2 is said to be a difference of two squares.

The fact that $(x+y)(x-y)=x^2-y^2$ means that $x+y$ and $x-y$ are factors of x^2-y^2.

Let us see whether we can find the factors of x^2-y^2 by considering it as a quadratic expression in x. We can rewrite it as a trinomial quadratic expression in x by writing the middle term as 0. We then obtain the expression x^2+0-y^2. Now consider its factorization.

The product of the first and last terms of the expression is $= x^2 \times (-y^2) = -x^2y^2$ and the middle term is 0.
Now,

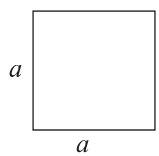
$$-x^2y^2 = (-xy) \times (xy) \text{ and } -xy+xy=0$$

Therefore, $x^2+0-y^2 = x^2-xy+xy-y^2$
$= x(x-y)+y(x-y)$
$= (x-y)(x+y)$

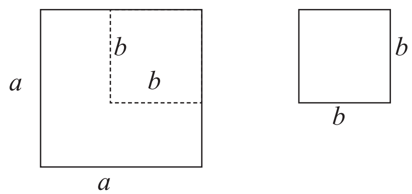
Again we obtain $x^2-y^2=(x-y)(x+y)$.

Now let us consider how to find the factors of the difference of two squares by considering areas.

Consider a square of side length a units.



From this square, cut out a square of side length b units.



The area of the remaining portion is a^2-b^2 square units.

Let us rearrange the remaining portion as follows.

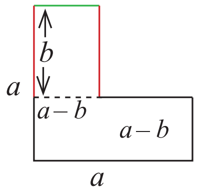


Figure I

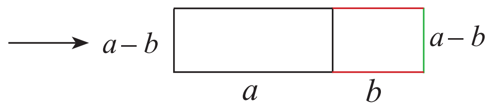


Figure II

The area of the remaining portion according to figure II is $(a-b)(a+b)$.
Accordingly, $a^2 - b^2 = (a-b)(a+b)$.

Now let us consider some examples of the factorization of expressions which are the difference of two squares.

Example 1

Factorize $x^2 - 25$.

$$\begin{aligned} x^2 - 25 &= x^2 - 5^2 \\ &= \underline{\underline{(x-5)(x+5)}} \end{aligned}$$

Example 2

Factorize $9 - y^2$.

$$\begin{aligned} 9 - y^2 &= 3^2 - y^2 \\ &= \underline{\underline{(3-y)(3+y)}} \end{aligned}$$

Example 3

Factorize $4a^2 - 49$.

$$\begin{aligned} 4a^2 - 49 &= 2^2a^2 - 7^2 \\ &= \underline{\underline{(2a-7)(2a+7)}} \end{aligned}$$

Example 4

Factorize $1 - 4b^2$.

$$\begin{aligned} 1 - 4b^2 &= 1^2 - 2^2b^2 \\ &= \underline{\underline{(1-2b)(1+2b)}} \end{aligned}$$

Example 5

Factorize $2x^2 - 72$.

$$\begin{aligned} 2x^2 - 72 &= 2\,(x^2 - 36) \\ &= 2\,(x^2 - 6^2) \\ &= \underline{\underline{2\,(x-6)(x+6)}} \end{aligned}$$

Example 6

Find the value $33^2 - 17^2$.

$$\begin{aligned} 33^2 - 17^2 &= (33 + 17)(33 - 17) \\ &= 50 \times 16 \\ &= \underline{\underline{800}} \end{aligned}$$

Example 7

Factorize $\frac{x^2}{4} - \frac{1}{9}$.

$$\frac{x^2}{4} - \frac{1}{9} = \frac{x^2}{2^2} - \frac{1}{3^2}$$

$$= \underline{\underline{\left(\frac{x}{2} + \frac{1}{3}\right)\left(\frac{x}{2} - \frac{1}{3}\right)}}$$

Example 8

Factorize $1 - \frac{9x^2}{16}$.

$$1 - \frac{9x^2}{16} = 1^2 - \left(\frac{3x}{4}\right)^2$$

$$= \underline{\underline{\left(1 - \frac{3x}{4}\right)\left(1 + \frac{3x}{4}\right)}}$$

Exercise 6.3

Factorize the expressions given below.

a. $x^2 - 100$ **b.** $m^2 - 36$ **c.** $p^2 - 81$

d. $4 - b^2$ **e.** $16 - a^2$ **f.** $64 - y^2$

g. $x^2 - 4y^2$ **h.** $9a^2 - 16b^2$ **i.** $100x^2 - 1$

j. $25m^2 - n^2$ **k.** $49 - 81p^2$ **l.** $25a^2b^2 - 9c^2$

Miscellaneous Exercise

1. Factorize the following algebraic expressions by changing the order in which the terms appear as required.

i. $ax + by - ay - bx$ **ii.** $9p - 2q - 6q + 3p$

iii. $x - 12 + x^2$ **iv.** $4 - k^2 - 3k$

2. Factorize the following algebraic expressions.

i. $8x^2 - 50$ **ii.** $3x^2 - 243$

iii. $a^3 b^3 - ab$ **iv.** $3 - 12q^2$

3. Find the value.

i. $23^2 - 3^2$ **ii.** $45^2 - 5^2$ **iii.** $102^2 - 2^2$

4. Join each algebraic expression in column A with the product of its factors in column B.

A	**B**
$x^2 - x - 6$	$\left(\frac{x}{5} - 1\right)\left(\frac{x}{5} + 1\right)$
$x^2 + 5x - 3x - 15$	$2x(x - 2)(x + 2)$
$2x^3 - 8x$	$(x - 3)(x + 5)$
$4x^2 - 9m^2$	$(x - 3)(x + 2)$
$\frac{x^2}{25} - 1$	$(2x - 3m)(2x + 3m)$

7 Axioms

By studying this lesson, you will be able to;

- identify five fundamental axioms of mathematics,
- develop geometrical relationships and solve problems involving calculations using the five fundamental axioms.

Axioms

Statements which are considered to be self-evident and are accepted without proof are called axioms. In mathematics, axioms are used to explain facts logically, develop relationships and reach conclusions.

Euclid, who is considered to be the father of geometry lived in Greece around 300 B.C. He introduced certain axioms related to mathematics in his book "Elements". Some of them are unique to geometry. Others are common axioms which can be used in other areas including algebra.

We consider five common axioms in this lesson. They can be summarized as given below.

1. Quantities which are equal to the same quantity, are equal.
2. Quantities which are obtained by adding equal quantities to equal quantities, are equal.
3. Quantities which are obtained by subtracting equal quantities from equal quantities, are equal.
4. Products which are equal quantities multiplied by equal quantities, are equal.
5. Quotients which are equal quantities divided by nonzero equal quantities, are equal.

By "quantities" we usually mean lengths, areas, volumes, masses, speeds, magnitudes of angles, etc.

These five axioms are very important because we can derive many results related to algebra and geometry by using them. Let us study these axioms in detail.

Axiom 1

Quantities which are equal to the same quantity, are equal.

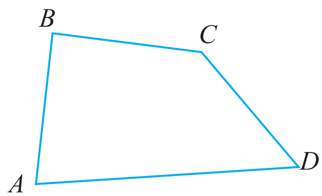
We can write this axiom briefly as given below.
If $b = a$ and $c = a$, then $b = c$.

According to this axiom,

'If Hasith's age is the same as Kasun's and Harsha's age is also the same as Kasun's, then Hasith's age is the same as Harsha's.'

How Axiom 1 is used to obtain geometrical results is seen in the simple example given below.

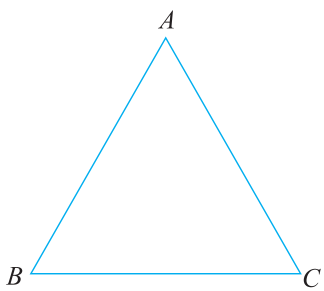
In the quadrilateral $ABCD$ shown below $BC = AB$ and $CD = AB$.



According to the above axiom,
$BC = CD$.

Example 1

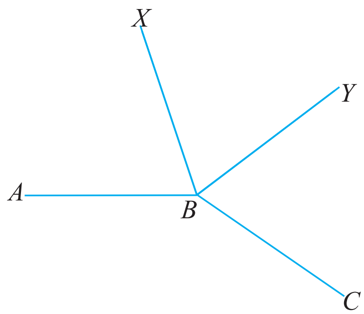
In the triangle ABC, $AB = AC$ and $AB = BC$ If $AC = 5$ cm then determine the perimeter of the triangle ABC.



Since $AC = 5$ cm and $AC = AB$, according to Axiom 1, $AB = 5$ cm.
Since $AB = 5$ cm and $AB = BC$, according to Axiom 1, $BC = 5$ cm.
The perimeter of the triangle $ABC = AC + BC + AB$
$= 5 \text{ cm} + 5 \text{ cm} + 5 \text{ cm}$
$= 15 \text{ cm}$

Example 2

In the figure given below, $X\hat{B}Y = A\hat{B}X$ and $X\hat{B}Y = C\hat{B}Y$. Find the relationship between $A\hat{B}X$ and $C\hat{B}Y$.

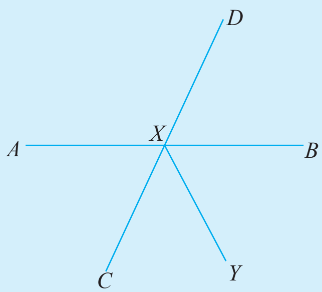


$$X\hat{B}Y = A\hat{B}X \text{ (given)}$$
$$X\hat{B}Y = C\hat{B}Y \text{ (given)}$$

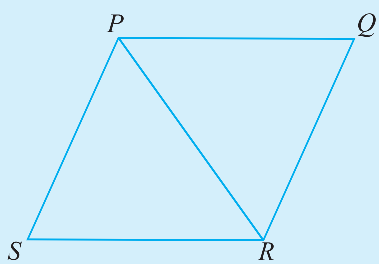
$\therefore$ According to Axiom 1, $A\hat{B}X = C\hat{B}Y$

Exercise 7.1

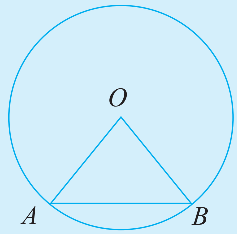
1. The straight lines AB and CD intersect at X. In the figure, $D\hat{X}B = B\hat{X}Y$. If $A\hat{X}C = 70^\circ$, find the magnitude of $B\hat{X}Y$.



2. In the parallelogram $PQRS$, $PQ = PR$ and $PQ = PS$. Based on its sides, mention what type of triangle PSR is.



3. The points A and B are located on the circle with centre O, such that $OA = AB$. Based on its sides, mention what type of triangle ABO is.



Axiom 2

Quantities which are obtained by adding equal quantities to equal quantities, are equal.

We can write this briefly as given below.

If $a = b$, then $a + c = b + c$.

This axiom can be written as given below too.

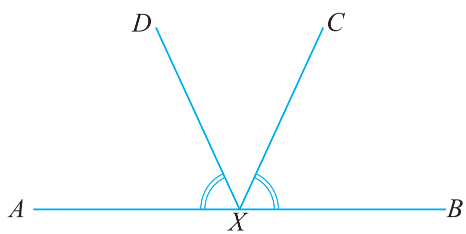
If $x = y$ and $p = q$, then $x + p = y + q$.

According to this axiom,

"If the cost incurred in purchasing vegetables is equal to the cost incurred in purchasing milk and the cost incurred in purchasing fruits is equal to the cost incurred in purchasing eggs, then the cost incurred in purchasing vegetables and fruits is equal to the cost incurred in purchasing milk and eggs."

Let us consider a simple geometrical result that can be derived using the above axiom.

In the figure given below, the point X is located on the straight line AB. Also, $A\hat{X}D = B\hat{X}C$.



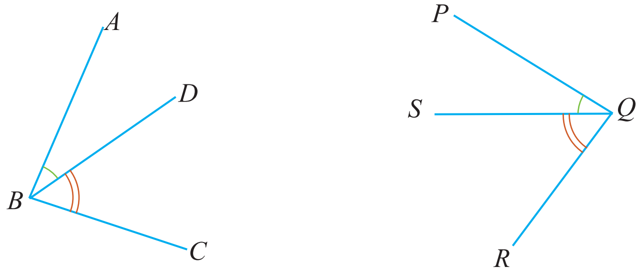
$$A\hat{X}D = B\hat{X}C \text{ (given)}$$

$\therefore$ According to Axiom 2, $\underbrace{A\hat{X}D + C\hat{X}D} = \underbrace{B\hat{X}C + C\hat{X}D}$

$$\therefore\ A\hat{X}C = B\hat{X}D$$

Example 1

In the figures given below, $A\hat{B}D = P\hat{Q}S$ and $C\hat{B}D = R\hat{Q}S$. Show that $A\hat{B}C = P\hat{Q}R$.



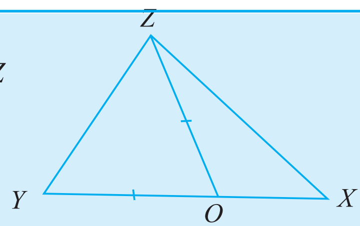
$A\hat{B}D = P\hat{Q}S$, $C\hat{B}D = R\hat{Q}S$.

$\therefore$ According to this axiom, $A\hat{B}D + C\hat{B}D = P\hat{Q}S + R\hat{Q}S$

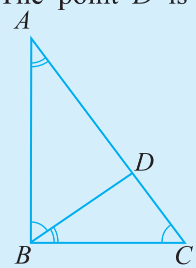
$\therefore\ A\hat{B}C = P\hat{Q}R$

Exercise 7.2

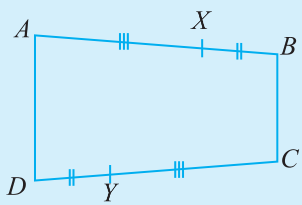
1. The point O is located on the side XY of the triangle XYZ such that $OZ = OY$. Show that $XY = OZ + OX$.



2. The point D is located on the side AC of the triangle ABC. If $A\hat{B}D = B\hat{C}D$ and $C\hat{B}D = B\hat{A}D$, show that $B\hat{A}D + B\hat{C}D = A\hat{B}C$.



3. The points X and Y are located on the sides AB and CD respectively of the quadrilateral $ABCD$, such that $AX = CY$ and $BX = DY$. Show that $AB = CD$.



Axiom 3

Quantities which are obtained by subtracting equal quantities from equal quantities, are equal.

We can write this briefly as given below.

If $a = b$, then $a - c = b - c$.

This axiom can be written as given below too.

If $a = b$ and $c = d$, then $a - c = b - d$.

A simple result in geometry that can be obtained by using the above axiom is given below.

In the figure given below, $AD = CB$.

$$AD = CB$$

According to Axiom 3, $AD - CD = CB - CD$

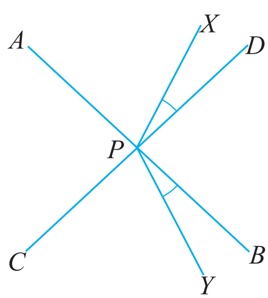
$$\therefore \; AC = DB$$

Example 1

The straight line segments AB and CD intersect at P. $X\hat{P}D = B\hat{P}Y$

(i) Show that $A\hat{P}X = C\hat{P}Y$.

(ii) If $A\hat{P}D = 95^{\circ}$ and $X\hat{P}D = 20^{\circ}$, find the magnitude of $C\hat{P}Y$.



(i) $A\hat{P}D = B\hat{P}C$ (vertically opposite angles)

$X\hat{P}D = B\hat{P}Y$ (given)

$\therefore$ According to this axiom, $\underbrace{A\hat{P}D - X\hat{P}D} = \underbrace{B\hat{P}C - B\hat{P}Y}$

$$\therefore \; A\hat{P}X = C\hat{P}Y$$

(ii) $A\hat{P}X = A\hat{P}D - X\hat{P}D$

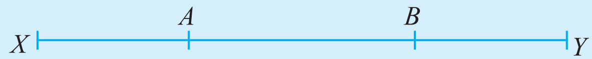
$A\hat{P}X = 95^{o} - 20^{o}$

$A\hat{P}X = 75^{o}$

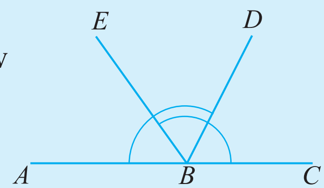
$\therefore\ C\hat{P}Y = 75^{o}$

Exercise 7.3

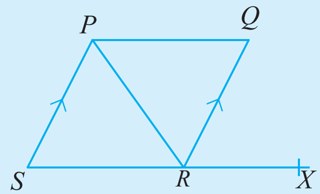
1. The points A and B are located on the line XY such that $XB = AY$. If $XY = 16$ cm and $BY = 6$ cm, find the length of AB.



2. The point B is located on the line AC. If $A\hat{B}D = C\hat{B}E$, show that $A\hat{B}E = C\hat{B}D$.



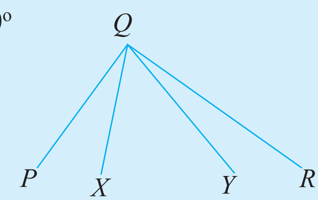
3. In the quadrilateral $PQRS$ in the figure, $S\hat{P}R = P\hat{R}Q$. If $Q\hat{P}S = P\hat{R}X$ and $S\hat{P}R = Q\hat{R}X$, show that $Q\hat{P}R = Q\hat{R}X$.



4. In the figure given here, $P\hat{Q}Y = X\hat{Q}R$. If $P\hat{Q}R = 110^{o}$ and $P\hat{Q}X = 35^{o}$,

(i) find the magnitude of $R\hat{Q}Y$.

(ii) find the magnitude of $X\hat{Q}Y$.



Axiom 4

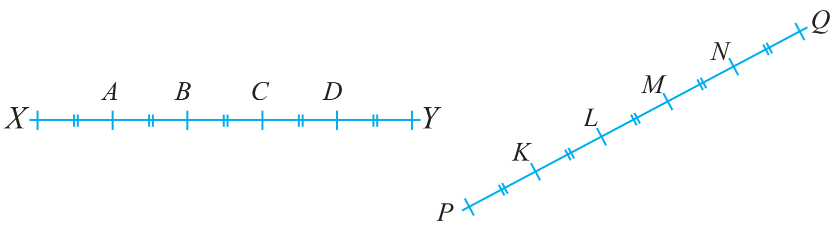
Products which are equal quantities multiplied by equals, are equal.

We can write this briefly as given below.

If $a = b$, then $ca = cb$.

Let us first consider an application of this axiom in geometry.

- As indicated in the figure given below, the points A, B, C and D are located on the straight line XY, such that $XA = AB = BC = CD = DY$. The points K, L, M and N are located on the straight line PQ, such that $PK = KL = LM = MN = NQ$. It is also given that $XA = PK$.



Let us show that $XY = PQ$.

Since $XA = AB = BC = CD = DY$, it is clear that $5\,XA = XY$.
Similarly, since $PK = KL = LM = MN = NQ$, we obtain that $5\,PK = PQ$.
Since $XA = PK$, by applying Axiom 4 we obtain $5\,XA = 5\,PK$.
$\therefore\ XY = PQ$.

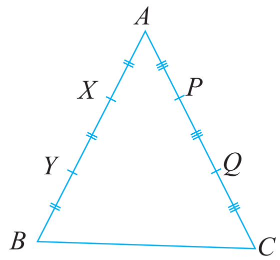
Although it is important to understand how results are derived by using axioms, in practice, when deriving geometrical results, the relevant axioms are not mentioned. This is because, as implied by the word "axiom", the validity of the derivation is obvious.

Now, let us consider how this axiom is applied in algebra.

If $x = 5$ and $y = 2x$, let us find the value of y.
Since $x = 5$, applying the above axiom and multiplying both sides by 2 we obtain $2x = 2 \times 5$.
But $2 \times 5 = 10$.
Hence by Axiom 1 we obtain
$y = 10$.

Example 1

The points X and Y are located on the side AB of the triangle ABC such that $AX = XY = YB$. The points P and Q are located on the side AC such that $AP = PQ = QC$. If $AX = AP$, find the relationship between AB and AC.



$AX = XY = YB$ (given)

$\therefore AB = 3AX$

$AP = PQ = QC$ (given)

$\therefore AC = 3AP$

$AX = AP$ (given)

According to Axiom 4;

$3AX = 3AP$

$\therefore AB = AC$

Axiom 5

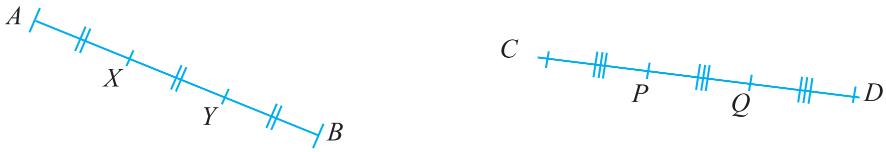
Quotients which are equal quantities divided by nonzero equals, are equal.

We can write this briefly as given below.

If $a = b$ then $\frac{a}{c} = \frac{b}{c}$.

Here, c is a nonzero number since it is meaningless to divide by zero.

- The line segments AB and CD in the figure are equal in length (that is, $AB = CD$). The points X and Y are located on AB such that $AX = XY = YB$. The points P and Q are located on CD such that $CP = PQ = QD$.



Let us show that $AX = CP$.

Since $AX = XY = YB$, we obtain that $\frac{AB}{3} = AX$.

Since $CP = PQ = QD$, we obtain that $\frac{CD}{3} = CP$.

Since $AB = CD$, according to Axiom 5,

$$\frac{AB}{3} = \frac{CD}{3}$$

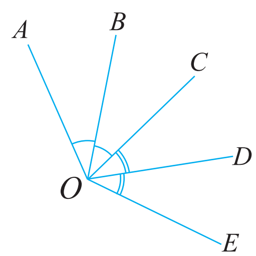
$\therefore AX = CP.$

Example 1

In the figure given below, $A\hat{O}B = B\hat{O}C$ and $C\hat{O}D = D\hat{O}E$. If $A\hat{O}C = C\hat{O}E,$

(i) find the relationship between $A\hat{O}B$ and $D\hat{O}E$.

(ii) If $B\hat{O}C = 35^\circ$, find the magnitude of $D\hat{O}E$.



(i) $A\hat{O}B = B\hat{O}C$ (given)

$\therefore A\hat{O}B = \dfrac{A\hat{O}C}{2}$

$C\hat{O}D = D\hat{O}E$ (given)

$\therefore D\hat{O}E = \dfrac{C\hat{O}E}{2}$

$A\hat{O}C = C\hat{O}E$ (given)

$\therefore$ According to Axiom 5, $\dfrac{A\hat{O}C}{2} = \dfrac{C\hat{O}E}{2}$

$\therefore \quad A\hat{O}B = D\hat{O}E$

(ii) $A\hat{O}B = B\hat{O}C$ (given) $\qquad B\hat{O}C = 35^\circ$ (given)

$\therefore A\hat{O}B = 35^\circ$ (by axiom 1)

$A\hat{O}B = D\hat{O}E$ (Proved above)

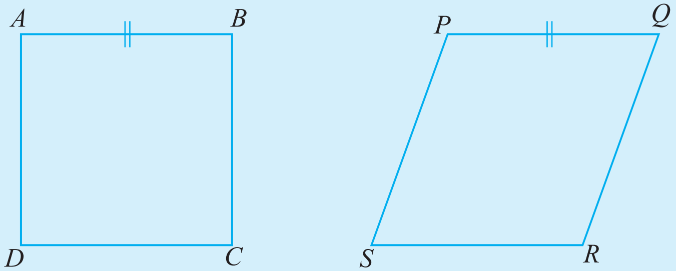
$\therefore D\hat{O}E = 35^\circ$ (by axiom 1)

Exercise 7.4

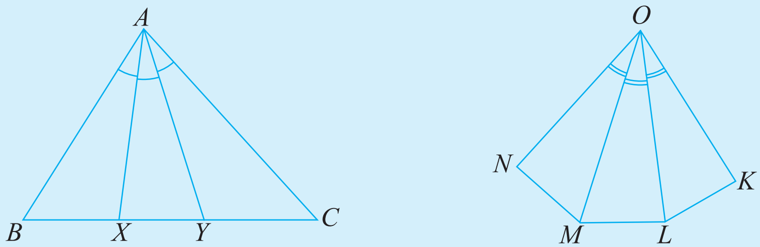
1. The square $ABCD$ and the rhombus $PQRS$ are such that $AB = PQ$.

(i) Using Axiom 4, show that the perimeter of the square $ABCD$ is equal to the perimeter of the rhombus $PQRS$.

(ii) If $AB = 7$cm, find the perimeter of the rhombus $PQRS$.



2.



In the triangle ABC, $B\hat{A}X = X\hat{A}Y = C\hat{A}Y$. In the pentagon $KLMNO$ $M\hat{O}N = L\hat{O}M = K\hat{O}L$. If $B\hat{A}C = K\hat{O}N$,

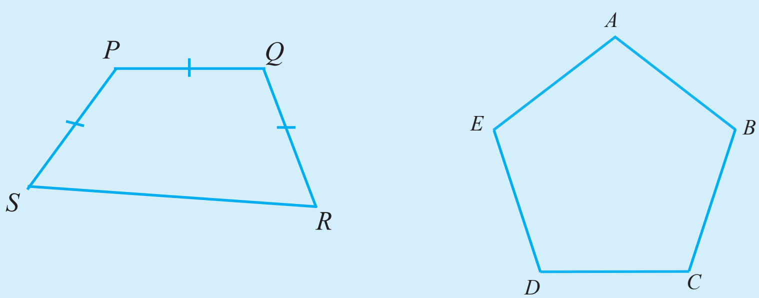
(i) show that $X\hat{A}Y = M\hat{O}L$.

(ii) If $X\hat{A}Y = 30^{o}$, determine the magnitude of $K\hat{O}N$.

3. In the quadrilateral $PQRS$, $PQ = QR = SP$ and $2PQ = RS$. The perimeter of the regular pentagon $ABCDE$ is equal to that of the quadrilateral $PQRS$.

(i) Find the relationship between PQ and AB.

(ii) If $AB = 8$ cm, find the perimeter of the quadrilateral $PQRS$.



Further applications of the axioms

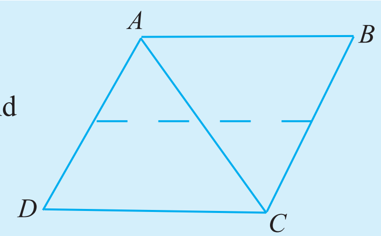
Example 1

Solve the equation given below using the axioms.
$2x + 5 = 13$

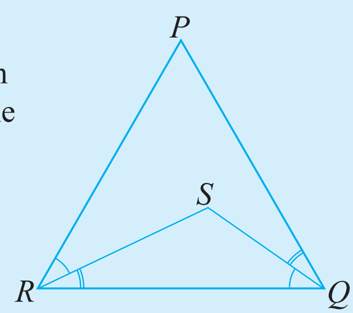
Here, solving the equation means finding the value of x. Since the quantity $2x + 5$ is equal to the quantity 13, according to Axiom 3, the quantities obtained by subtracting 5 from these two quantities are also equal.
$\therefore\ 2x + 5 - 5 = 13 - 5.$
Simplifying this we obtain,
$2x = 8.$
Since the quantity $2x$ is equal to the quantity 8, the quantities that are obtained by dividing these two quantities by 2 are also equal. Therefore, $\frac{2x}{2} = \frac{8}{2}$.

When we simplify this we obtain $x = 4$. That is, the solution of the equation is 4.

Miscellaneous Exercise



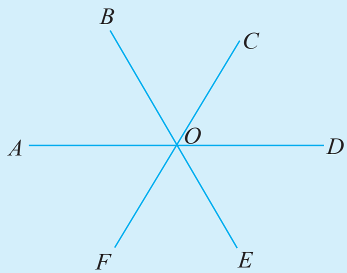
1. In the quadrilateral $ABCD$, $AD = AC$, $BC = AC$, $AB = BC$ and $AD = CD$. Show that $ABCD$ is a rhombus.



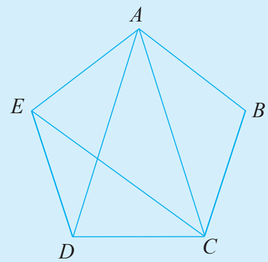
2. As indicated in the figure, the point S is located such that $P\hat{R}S = S\hat{Q}R$ and $Q\hat{R}S = P\hat{Q}S$. By applying the axioms,

(i) show that $P\hat{R}Q = P\hat{Q}R$,
(ii) show that if $R\hat{P}Q = P\hat{R}Q$, then all the angles of the triangle PQR are equal in magnitude.

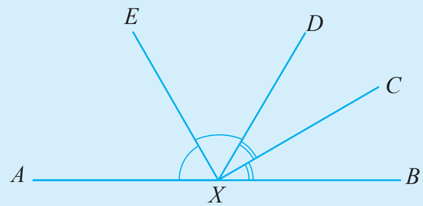
3. As indicated in the figure, the straight lines AD, BE and CF intersect at the point O. If $D\hat{O}E = A\hat{O}F$, show that $B\hat{O}D = D\hat{O}F$.



4. In the regular pentagon $ABCDE$, $E\hat{A}D = D\hat{A}C = B\hat{A}C$ and $B\hat{C}A = A\hat{C}E = D\hat{C}E$.
 (i) Show that $B\hat{C}A = B\hat{A}C$.
 (ii) If $B\hat{A}C = 36^{\circ}$ find the magnitude of $C\hat{D}E$.



5. The point X lies on the straight line AB. Also, $A\hat{X}E = E\hat{X}D$ and $B\hat{X}C = C\hat{X}D$. Determine the magnitude of $C\hat{X}E$.



Summary

Summary

- Quantities which are equal to the same quantity, are equal.
- Quantities which are obtained by adding equal quantities to equal quantities, are equal.
- Quantities which are obtained by subtracting equal quantities from equal quantities, are equal.
- Products which are equal quantities multiplied by equal quantities, are equal.
- Quotients which are equal quantities divided by nonzero equal quantities, are equal.

8 Angles related to straight lines and parallel lines

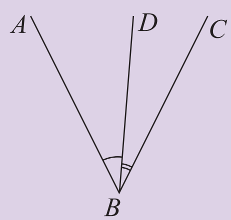
By studying this lesson, you will be able to;

- identify and verify the theorems related to the adjacent angles/vertically opposite angles formed by one straight line meeting or intersecting another straight line, and use them to solve problems,
- identify the angles formed when a transversal intersects two straight lines,
- identify and verify the theorems related to the angles formed when a transversal intersects two straight lines, and use them to solve problems.

Introduction

Let us first recall the basic geometrical facts we learnt in previous grades.

Adjacent angles



The angles $A\hat{B}D$ and $D\hat{B}C$ in the above figure have a common vertex. This common vertex is B. They also have a common arm BD. The pair of angles $A\hat{B}D$ and $D\hat{B}C$ lie on opposite sides of the common arm BD. Such a pair of angles is known as a pair of adjacent angles.

$A\hat{B}D$ and $D\hat{B}C$ are a pair of adjacent angles.

However, $A\hat{B}D$ and $A\hat{B}C$ are not a pair of adjacent angles. This is because, these two angles are not on opposite sides of the common arm AB.

Complementary angles

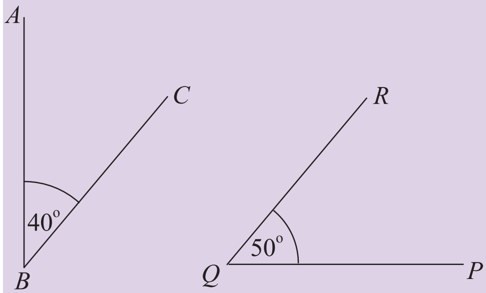


Figure I

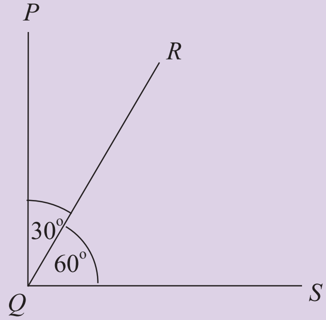


Figure II

In figure I, since $A\hat{B}C + P\hat{Q}R = 40^\circ + 50^\circ = 90^\circ$, $A\hat{B}C$ and $P\hat{Q}R$ are a pair of complementary angles.

In figure II, $P\hat{Q}R$ and $R\hat{Q}S$ are a pair of adjacent angles. Furthermore, since $P\hat{Q}R + R\hat{Q}S = 90^\circ$, they are a pair of complementary angles too. Therefore, $P\hat{Q}R$ and $R\hat{Q}S$ are a pair of complementary adjacent angles.

Supplementary angles

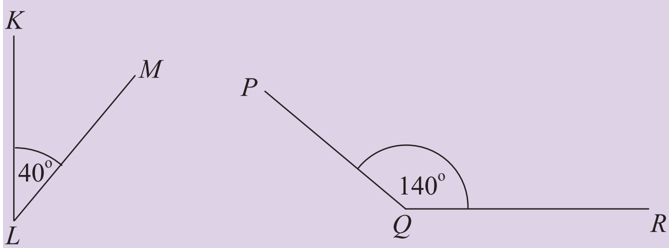


Figure I

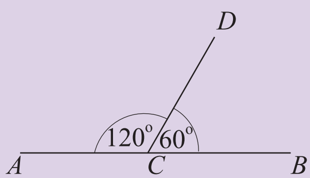
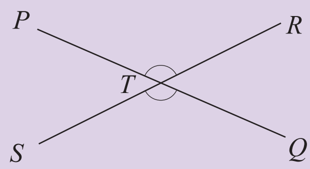


Figure II

In figure I, since $K\hat{L}M + P\hat{Q}R = 180^\circ$, $K\hat{L}M$ and $P\hat{Q}R$ are a pair of supplementary angles. In figure II, $A\hat{C}D$ and $B\hat{C}D$ are a pair of adjacent angles. Furthermore, since $A\hat{C}D + B\hat{C}D = 180^\circ$, they are a pair of supplementary angles too. Therefore, $A\hat{C}D$ and $B\hat{C}D$ are a pair of supplementary adjacent angles.

Vertically opposite angles



The pair of angles $P\hat{T}R$ and $S\hat{T}Q$, formed by the intersection of the straight lines PQ and RS at the point T, are vertically opposite angles.

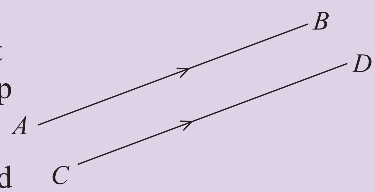
Similarly, $P\hat{T}S$ and $R\hat{T}Q$ are another pair of vertically opposite angles.

Vertically opposite angles are equal in magnitude.

Therefore, $P\hat{T}R = S\hat{T}Q$ and $P\hat{T}S = R\hat{T}Q$.

Parallel lines

Two straight lines in a plane which do not intersect each other are called parallel straight lines. The gap between two parallel straight lines is a constant.

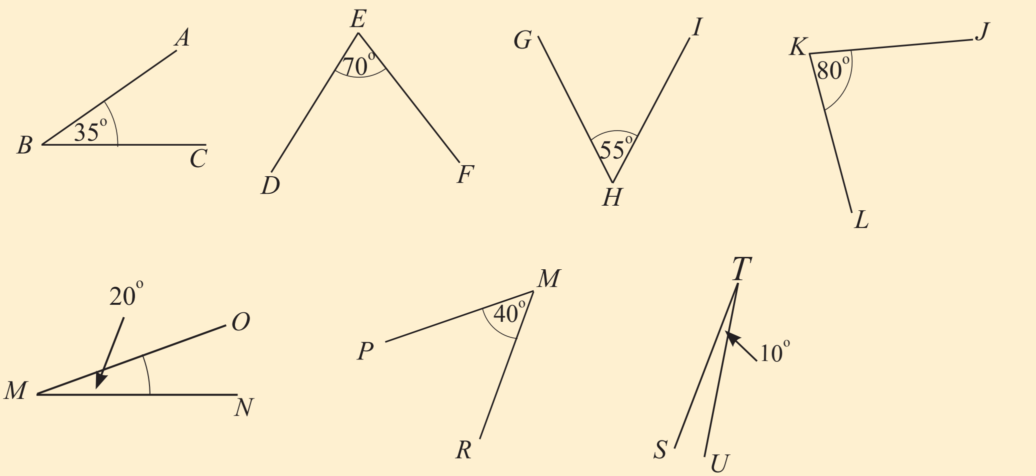


As shown in the figure, parallel lines are indicated using arrow. We use the notation $AB//CD$ to indicate that AB and CD are Parallel.

Do the following exercise, to strengthen your understanding of the above facts.

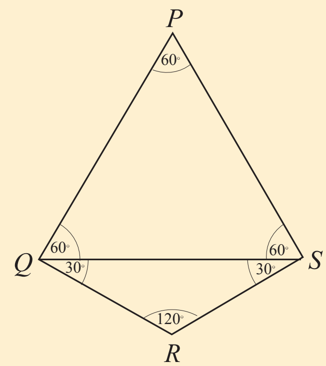
Review Exercise

1. From the angles given below, select and write the pairs which are complementary.

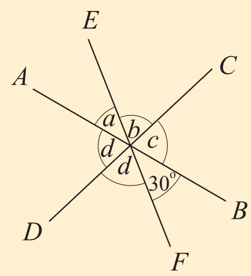


2. Based on the magnitudes of the angles shown in the figure, write

i. four pairs of complementary angles,
ii. two pairs of complementary adjacent angles,
iii. two pairs of supplementary angles.

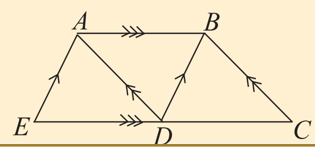


3. The straight line segments AB, CD and EF shown in the figure intersect at a point. According to the information given in the figure,



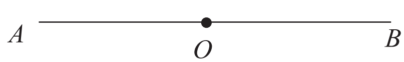
i. find the value denoted by a.
ii. give reasons why $b = d$.
iii. find the value denoted by d.
iv. find the values denoted by b and c.

4. Name three pairs of parallel straight lines.

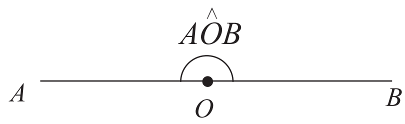


8.1 Angles related to straight lines

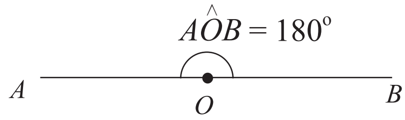
Let us assume that the point O is located on the straight line AB.



Then $A\hat{O}B$ can be considered as an angle between the arms AO and OB. Such an angle is known as a straight angle.

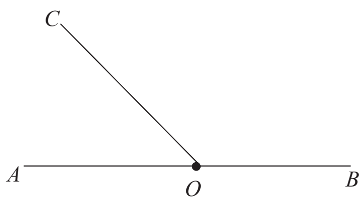


The unit “degree” which is used to measure angles has been selected such that a straight angle is equal in magnitude to 180°. Therefore, we can write $A\hat{O}B = 180^{\circ}$.



Accordingly, the magnitude of a straight angle is 180°.

In the following figure, two angles have been drawn at the point *O* which is located on the straight line *AB*.



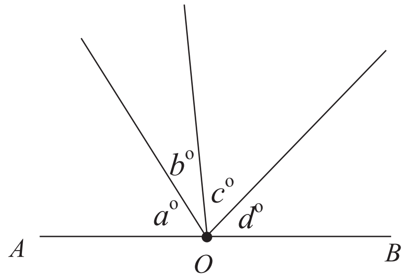
Here, $A\hat{O}C$ and $B\hat{O}C$ are a pair of adjacent angles. In a situation such as this, we say that the adjacent angles $A\hat{O}C$ and $B\hat{O}C$ are on the straight line *AB*. Furthermore, since $A\hat{O}B = 180^{\circ}$, it is clear that,

$$A\hat{O}C + B\hat{O}C = 180^{\circ}.$$

Hence, the two angles $A\hat{O}C$ and $B\hat{O}C$ are a pair of supplementary adjacent angles. The facts we have discussed can be stated as a theorem as follows.

Theorem:
The sum of the adjacent angles formed by a straight line meeting another straight line is two right angles.

The facts discussed above can be generalized further. As an example, in the figure given below, four angles have been drawn at the point *O* on the straight line *AB*.



The values of these angles in degrees have been denoted by a, b, c and d. In a situation such as this too, we say that the angles are on the straight line AB. Furthermore, since $A\hat{O}B = 180^o$, it is clear that,

$$a + b + c + d = 180.$$

It is also clear that this relationship holds true for any number of angles on a straight line. Therefore,

"The sum of the magnitudes of the angles on a straight line is 180°."

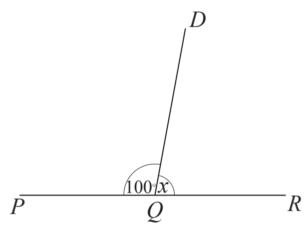
Now, by considering some examples, let us learn how to solve problems using these theorems.

Example 1

In each figure given below, if PQR is a straight line, then find the value denoted by x.

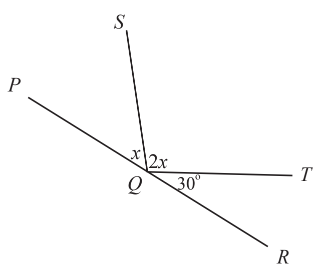
$P\hat{Q}D + D\hat{Q}R = 180^o$ (Angles on the straight line PQR)

$$100^o + x = 180^o$$
$$x = 180^o - 100^o$$
$$= \underline{\underline{80^o}}$$



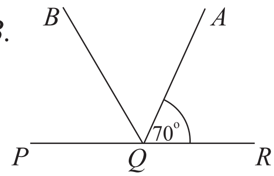
$P\hat{Q}S + S\hat{Q}T + T\hat{Q}R = 180^o$ (Angles on the straight line PQR)

$$x + 2x + 30^o = 180^o$$
$$3x + 30^o = 180^o$$
$$3x = 180^o - 30^o$$
$$3x = 150^o$$
$$x = \underline{\underline{50^o}}$$



Example 2

In the given figure, $A\hat{Q}R = 70^\circ$ and the bisector of $P\hat{Q}A$ is QB. If PQR is a straight line, then find the magnitude of $A\hat{Q}B$.



Since PQR is a straight line,

$$\begin{aligned} P\hat{Q}A + A\hat{Q}R &= 180^\circ \quad \text{(angles on the straight line } PQR\text{)} \\ P\hat{Q}A + 70^\circ &= 180^\circ \\ \therefore \quad P\hat{Q}A &= 180^\circ - 70^\circ \\ &= 110^\circ \end{aligned}$$

Since BQ is the bisector of $P\hat{Q}A$,

$$P\hat{Q}B = A\hat{Q}B = \frac{1}{2}\, P\hat{Q}A$$

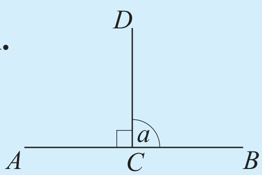
$$\therefore \quad A\hat{Q}B = \frac{110^\circ}{2}$$

$$= \underline{\underline{55^\circ}}$$

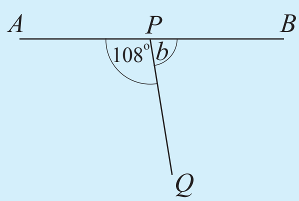
Exercise 8.1

1. In each of the following figures, AB is a straight line. Based on the information in each figure, find the value of the angle denoted by the lower case letter.

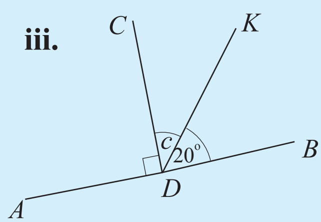
i.



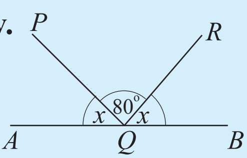
ii.



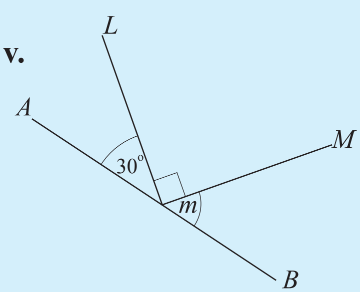
iii.



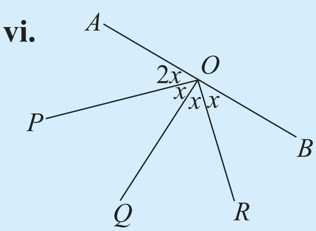
iv.



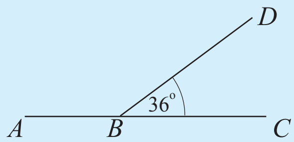
v.



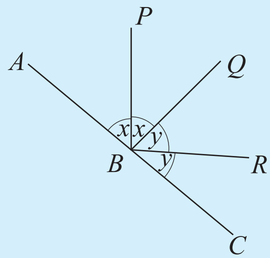
vi.



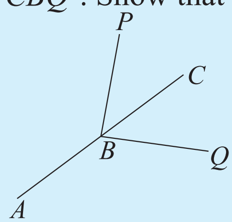
2. In the figure, ABC is a straight line. If $D\hat{B}C = 36^\circ$, show that the magnitude of $A\hat{B}D$ is four times the magnitude of $D\hat{B}C$.



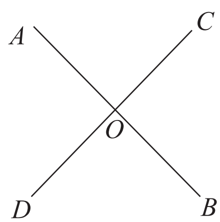
3. In the figure, ABC is a straight line. Based on the information given in the figure, show that $P\hat{B}R$ is a right angle.



4. In the figure, ABC is a straight line. Moreover, $P\hat{B}C = C\hat{B}Q$. Show that $A\hat{B}P = A\hat{B}Q$.



8.2 Vertically opposite angles



In the figure, the straight lines AB and CD intersect each other at O.

The vertex O is common to both the angles $A\hat{O}C$ and $D\hat{O}B$. Furthermore, they are on opposite sides of O.

The pair of angles $A\hat{O}C$ and $D\hat{O}B$ are known as a pair of vertically opposite angles.

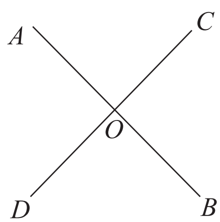
Similarly, $A\hat{O}D$ and $B\hat{O}C$ lie on opposite sides of O, which is the common vertex of these two angles.

Therefore $A\hat{O}D$ and $B\hat{O}C$ are also a pair of vertically opposite angles.

Accordingly, it is clear that two pairs of vertically opposite angles are formed by the intersection of two straight lines.
Let us now consider a theorem related to vertically opposite angles.

Theorem:
The vertically opposite angles formed by the intersection of two straight lines are equal.

By considering the figure, it can clearly be seen that 'vertically opposite angles are equal'. However, by using the fact we learnt earlier, that 'the sum of the angles on a straight line is 180', and also the axioms learnt in the previous lesson, this theorem can be proved as follows.



Data: The straight lines AB and CD intersect each other at O.
To Prove: $A\hat{O}C = B\hat{O}D$ and

$$A\hat{O}D = B\hat{O}C$$

Proof:
Since AB is a straight line,

$$A\hat{O}C + B\hat{O}C = 180^{\circ} \text{ (angles on the straight line } AB\text{)}$$

Similarly, since CD is a straight line,

$$B\hat{O}C + B\hat{O}D = 180^{\circ} \text{ (angles on the straight line } CD\text{)}$$

$$\therefore\ A\hat{O}C + B\hat{O}C = B\hat{O}C + B\hat{O}D \text{ (axiom)}$$

Subtracting $B\hat{O}C$ from both sides of the equation,

$$A\hat{O}C + \cancel{B\hat{O}C} - \cancel{B\hat{O}C} = \cancel{B\hat{O}C} - \cancel{B\hat{O}C} + B\hat{O}D$$

$$\therefore\ A\hat{O}C = B\hat{O}D$$

Similarly, $A\hat{O}D + A\hat{O}C = 180^{\circ}$ (angles on the straight line CD)

$\hat{AOC} + \hat{BOC} = 180^\circ$ (since AB is a straight line)

$\therefore \quad \hat{AOD} + \hat{AOC} = \hat{AOC} + \hat{BOC}$ (axiom)

Subtracting $\hat{AOC}$ from both sides of the equation we obtain,

$\hat{AOD} = \hat{BOC}$.

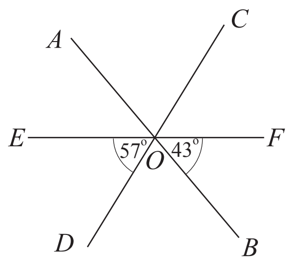
Let us consider the following examples to learn how this theorem is used to solve problems.

Example 1

Based on the information given in the figure, giving reasons, determine the following.

(i) The magnitude of $\hat{DOB}$

(ii) The magnitude of $\hat{AOC}$.



(i) Since EOF is a straight line,

$\hat{EOD} + \hat{DOB} + \hat{BOF} = 180^\circ$ (sum of the magnitudes of the angles on a straight line)

$57^\circ + \hat{DOB} + 43^\circ = 180^\circ$

$\hat{DOB} = 180^\circ - (57^\circ + 43^\circ)$

$\therefore \quad \hat{DOB} = 80^\circ$

(ii) $\hat{AOC} = \hat{DOB}$ (vertically opposite angles)

$\hat{DOB} = 80^\circ$ (proved above)

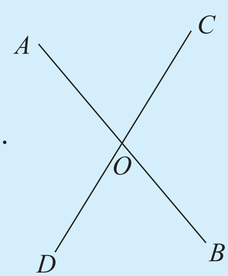
$\therefore \quad \hat{AOC} = \underline{\underline{80^\circ}}$

Exercise 8.2

1. In the figure, the straight lines AB and CD intersect each other at O.

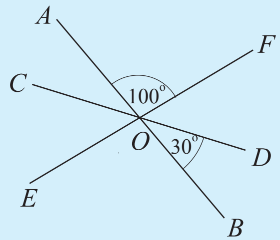
i. If $\hat{AOC} = 80^\circ$, find the magnitude of $\hat{BOD}$.

ii. Name an angle which is equal in magnitude to $\hat{AOD}$.



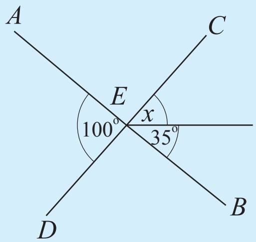
2. In the figure, the straight lines AB, CD and EF intersect at O. Based on the information provided in the figure, find the magnitude of each of the following angles.

i. $A\hat{O}C$
ii. $B\hat{O}E$
iii. $C\hat{O}E$



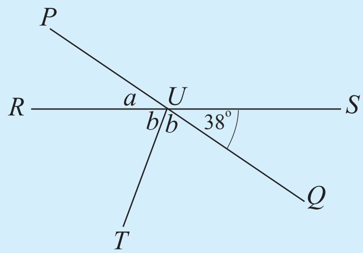
3. Based on the information given in each of the figures shown below, find the value of each English letter representing an angle.

i.



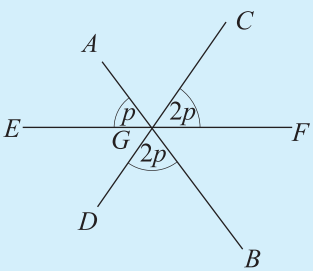
In the figure AB and CD are straight lines.

ii.



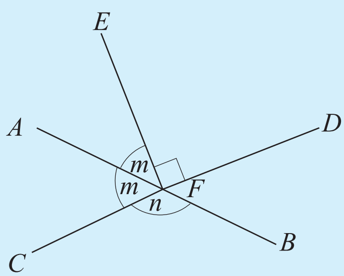
In the figure, RS and PQ are straight lines.

iii.



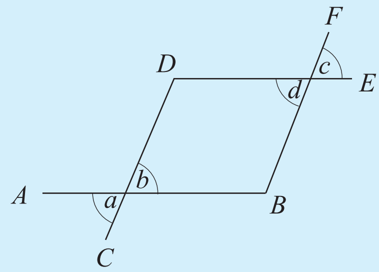
In the figure, the straight lines AB, CD and EF intersect at G.

iv.



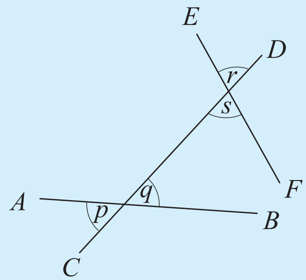
In the figure, AB and CD are straight lines.

4. In the figure, AB, CD, DE and BF are straight lines. Moreover, $a = d$. Fill in the blanks given below to prove that $b = c$.

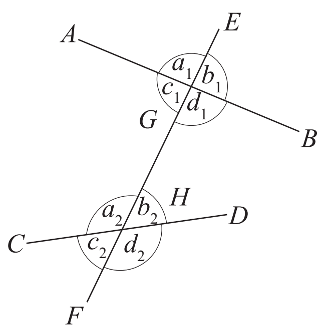


$a = b$ (................)
$d =$ ..... (................)
But, ... = ... (data)
$\therefore$ $b = c$

5. In the figure, AB, CD and EF are straight lines. Moreover $p = r$. Prove that $s = q$.



8.3 Corresponding angles, alternate angles and allied angles



In the above figure, the two straight line AB and CD are intersected by the straight line EF at the points G and H respectively. The line EF is known as a transversal.

A line intersecting two or more straight lines is known as a transversal.

In the above figure, there are four angles around the point G and four angles around the point H. According to where these angles are located, they are given special names in pairs.

Corresponding angles

Consider the four pairs of angles given below.

(i) a_1 and a_2 (ii) b_1 and b_2 (iii) c_1 and c_2 (iv) d_1 and d_2

Each of these pairs of angles is a pair of corresponding angles.

To be a pair of corresponding angles, the following characteristics should be there in the two angles.

1. Both angles should lie on the same side of the transversal.

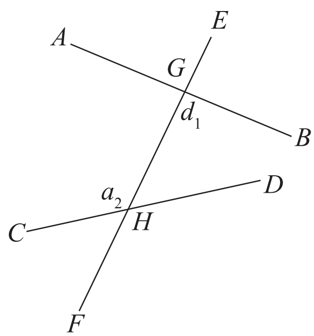
According to the above figure, both the angles a_1 and a_2 are on the left side of the transversal. Similarly, both the angles b_1 and b_2 are on the right side of the transversal. Also, the two angles c_1 and c_2 are on the left side of the transversal and the two angles d_1 and d_2 are on the right side of the transversal.

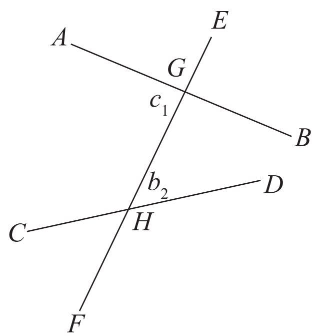
2. Both angles should be in the same direction with respect to the two straight lines.

According to the given figure, the angles a_1 and a_2 lie above the lines AB and CD respectively. The angles b_1 and b_2 also lie above the lines AB and CD respectively. Similarly, the angles c_1 and c_2 lie below the lines AB and CD respectively, and the angles d_1 and d_2 also lie below the lines AB and CD respectively.

In the given figure, the pairs of angles $A\hat{G}E$ and $C\hat{H}G$, $B\hat{G}E$ and $D\hat{H}G$, $A\hat{G}H$ and $C\hat{H}F$, $B\hat{G}H$ and $D\hat{H}F$ are pairs of corresponding angles.

Alternate angles





In the figures, the two pairs of angles given below are pairs of alternate angles.

(i) a_2 and d_1

(ii) c_1 and b_2

The characteristics common to these pairs of angles that can be used to identify pairs of alternate angles are the following.

1. The two angles should be on opposite sides of the transversal.

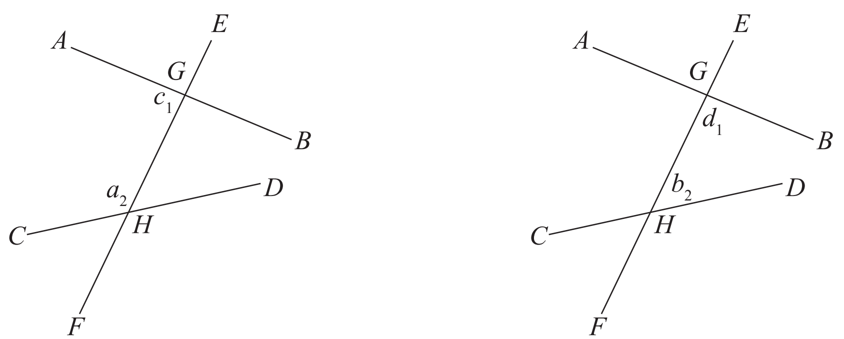
According to the above figure, the angles a_2 and d_1 lie on opposite sides of the transversal. Similarly, c_1 and b_2 also lie on opposite sides of the transversal.

2. The line segment of the transversal, which lies between the two straight lines, should be a common arm of the two angles.

According to the given figure, the line segment GH is a common arm of the angles a_2 and d_1 and also of the angles c_1 and b_2.

In the given figure, the pair of angles $B\hat{G}H$ and $G\hat{H}C$ and the pair of angles $A\hat{G}H$ and $G\hat{H}D$ are pairs of alternate angles.

Allied Angles



In the figures, the two pairs of angles given below are pairs of allied angles.

(i) a_2 and c_1
(ii) d_1 and b_2

In the figure, the two straight lines are intersected by a transversal. The pairs of angles on the same side of the transversal segment GH, between the straight lines AB and CD are

(i) the pair $A\hat{G}H$ and $C\hat{H}G$
(ii) the pair $B\hat{G}H$ and $D\hat{H}G$

For all four of these angles, GH is a common arm. A pair of angles on the same side of the common arm GH and between the straight lines AB and CD is called a pair of allied angles, Accordingly,

while the pair of angles $A\hat{G}H$ and $C\hat{H}G$ is a pair of allied angles, the pair of angles $B\hat{G}H$ and $D\hat{H}G$ is also a pair of allied angles.

Exercise 8.3

1. Consider the figures given below of two straight lines intersected by a transversal.

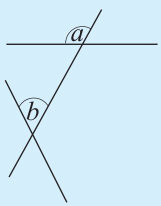


Figure 1

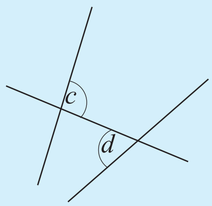


Figure 2

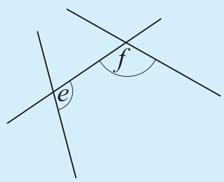
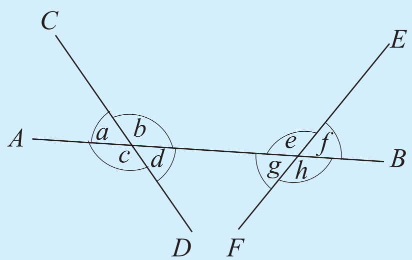


Figure 3

By considering the angles represented by the lower case English letters in the given figures, fill in the blanks.

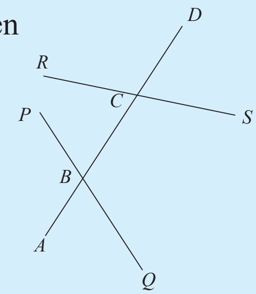
(i) In figure 1, a and b form a pair of .... angles.
(ii) In figure 2, c and d form a pair of .... angles.
(iii) In figure 3, e and f form a pair of .... angles.

2. Consider the figure given below of two straight lines intersected by a transversal. Its angles are indicated by lowercase English letters.



i. Name the line which can be considered as the transversal.
ii. Name the two straight lines which are intersected by the transversal.
iii. One pair of corresponding angles is the pair of angles a and e. Write the other three pairs of corresponding angles in a similar manner.
iv. Write the two pairs of allied angles in terms of the lower case English letters.
v. Write the two pairs of alternate angles in terms of the lower case English letters.

3. Answer the questions given below in relation to the given figure of two straight lines intersected by a transversal.



(i) Name the angle which together with $A\hat{B}P$ forms a pair of corresponding angles.

(ii) Name the angle which together with $B\hat{C}S$ forms the following.

a) Pair of allied angles

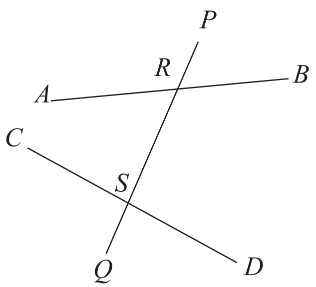
b) Pair of alternate angles

c) Pair of corresponding angles.

(iii) What type of angles is the pair $R\hat{C}D$ and $P\hat{B}C$?

(iv) What type of angles is the pair $P\hat{B}C$ and $B\hat{C}R$?

8.4 Angles related to parallel lines



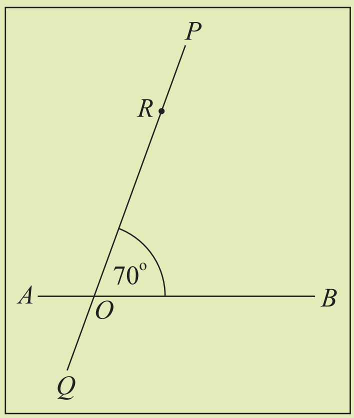
As indicated in the figure, the transversal PQ intersects the two straight lines AB and CD at R and S respectively. Now let us consider how the two lines AB and CD are positioned in each of the following cases.

- ⋆ When a pair of corresponding angles are equal
- ⋆ When a pair of alternate angles are equal
- ⋆ When the sum of a pair of allied angles is 180°

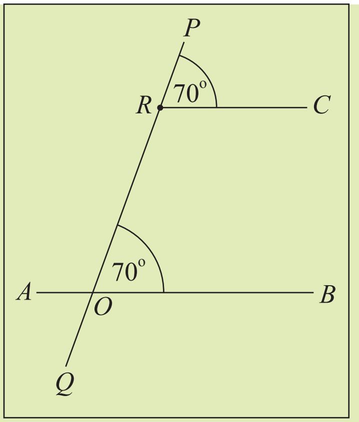
Do the following activity to identify this.

Activity 1

Step 1: On a sheet of A4 paper, draw two straight lines AB and PQ such that they intersect at O and such that $P\hat{O}B = 70^{\circ}$, as shown in the figure. Mark a point R on OP.



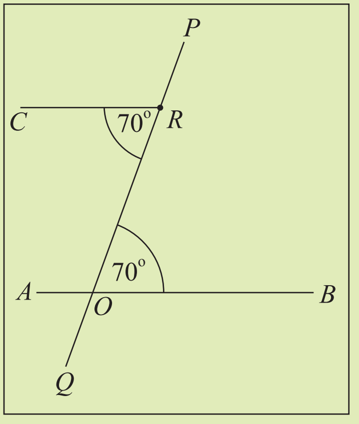
Step 2: Using the protractor, as shown in the figure, draw $P\hat{R}C$ at R such that its magnitude is 70°. Observe that $P\hat{O}B$ and $P\hat{R}C$ are a pair of corresponding angles (considering PQ as a transversal which intersects the straight lines RC and AB)



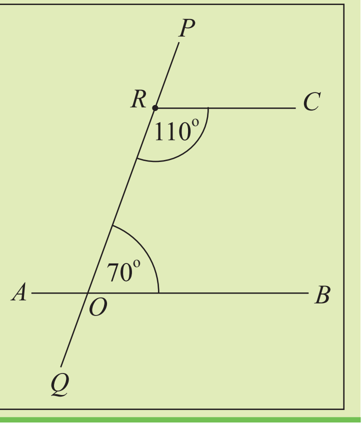
Step 3: Using a set square and a straight edge, examine whether the lines AB and RC are parallel.

Step 4: Selecting different values for $P\hat{O}B$, repeat the above three steps and in each case examine whether the lines AB and RC are parallel.

Step 5: Carry out steps 1 to 3 which were performed for corresponding angles, for alternate angles too. When completing these steps you will obtain a figure similar to the one shown here.



Step 6: Carry out the steps which were performed for corresponding angles, for allied angles too. In this case, the line drawn in step 2 above should be drawn as shown in the figure, such that $C\hat{R}O = 180^{\circ} - 70^{\circ} = 110^{\circ}$.



In doing the above activity you would have observed that, when

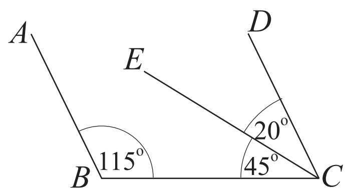
(i) a pair of corresponding angles are equal or
(ii) a pair of alternate angles are equal or
(iii) the sum of a pair of allied angles is equal to 180^{o},

then the straight lines *AB* and *RC* are parallel.

This result which is true in general, can be expressed as a theorem as follows.

Theorem : When two straight lines are intersected by a transversal, if
(i) a pair of corresponding angles are equal or
(ii) a pair of alternate angles are equal or
(iii) the sum of a pair of allied angles is 180^{o}, then the two straight lines are parallel to each other.

Example 1



Based on the information given in the figure, show that *AB* and *DC* are parallel. The two angles $A\hat{B}C$ and $B\hat{C}D$ formed by the transversal *BC* meeting the two straight lines *AB* and *DC*, are a pair of allied angles.

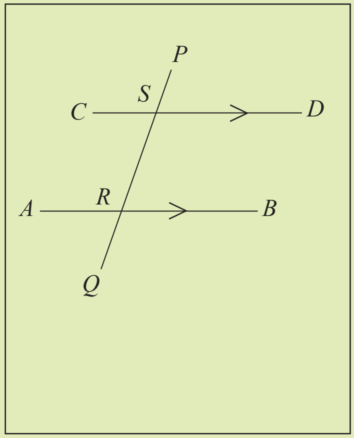
$$A\hat{B}C = 115^{o}$$
$$B\hat{C}D = B\hat{C}E + E\hat{C}D = 45^{o} + 20^{o} = 65^{o}$$
$$\therefore \quad A\hat{B}C + B\hat{C}D = 115^{o} + 65^{o} = 180^{o}$$

Since the sum of the pair of allied angles $A\hat{B}C$ and $B\hat{C}D$ is 180^{o} , *AB* and *DC* are parallel.

Now let us consider another theorem which is related to parallel lines.

Activity 2

Step 1: On a sheet of A4 paper, draw two straight lines AB and CD parallel to each other (the parallel lines can be drawn using a set square and a straight edge), and a transversal PQ such that it intersects the lines AB and CD at R and S respectively, as shown in the figure.



Step 2: Use a protractor to measure the magnitudes of the below given angles.

(i) Measure the magnitudes of the pair of corresponding angles $S\hat{R}B$ and $P\hat{S}D$ and check whether they are equal. Similarly, measure the magnitudes of the other pairs of corresponding angles and check whether they too are equal.

(ii) Measure the magnitudes of the pair of alternate angles $C\hat{S}R$ and $S\hat{R}B$ and check whether they are equal. Similarly, measure the magnitudes of the other pair of alternate angles and check whether they too are equal.

(iii) Measure the magnitudes of the pair of allied angles $D\hat{S}R$ and $S\hat{R}B$ and check whether they are supplementary. Similarly, measure the magnitudes of the other pair of allied angles and check whether they too are supplementary.

Step 3: Change the inclination of the transversal PQ and repeat the above two steps.

You would have observed in the above activity that when two parallel lines are intersected by a transversal,

(i) each pair of corresponding angles is equal,
(ii) each pair of alternate angles is equal,
(iii) each pair of allied angles is supplementary.

This result is true in general and can be expressed as a theorem as follows.

Theorem: When a transversal intersects a pair of parallel lines,

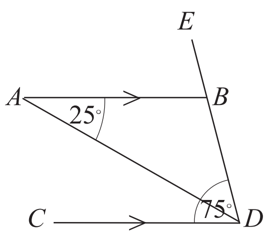
i. the corresponding angles formed are equal,

ii. the alternate angles formed are equal,

iii. the sum of each pair of allied angles formed equals two right angles.

Observe that the above theorem is the converse of the theorem learnt earlier.

Example 2



In the above figure, the straight lines AB and CD are parallel (this is denoted by $AB//CD$). Moreover, $B\hat{D}C = 75^\circ$ and $B\hat{A}D = 25^\circ$.

(i) Giving reasons, determine the magnitude of $A\hat{B}E$.

(ii) Giving reasons, determine the magnitude of $A\hat{D}B$.

(i)

$B\hat{D}C = 75^\circ$ (data)

$B\hat{D}C = A\hat{B}E$ (corresponding angles, $AB//CD$)

$\therefore \quad A\hat{B}E = \underline{\underline{75^\circ}}$

(ii)

$B\hat{A}D = 25^\circ$ (data)

$B\hat{A}D = A\hat{D}C$ (alternate angles, $AB//CD$)

$\therefore \quad A\hat{D}C = 25^\circ$

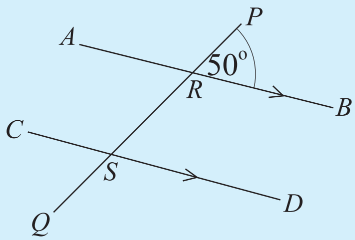
But $\quad A\hat{D}B = B\hat{D}C - A\hat{D}C$

$= 75^\circ - 25^\circ$

$= \underline{\underline{50^\circ}}$

Exercise 8.4

1.

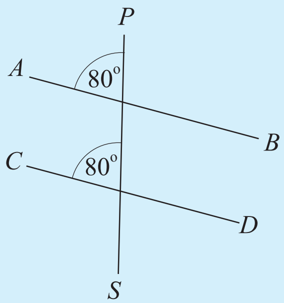


In the figure, $AB//CD$. If $P\hat{R}B = 50^{\circ}$, find the magnitude of each of the following angles.

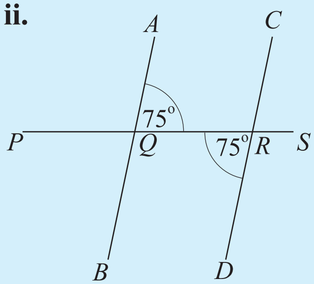
i. $R\hat{S}D$ (ii) $A\hat{R}S$ (iii) $C\hat{S}Q$ (iv) $Q\hat{S}D$

2. Based on the information in each of the following figures, giving reasons state whether AB and CD are parallel.

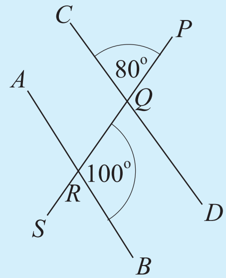
i.



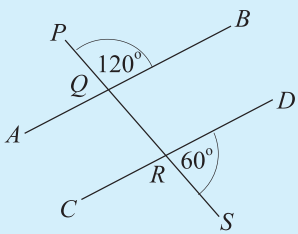
ii.



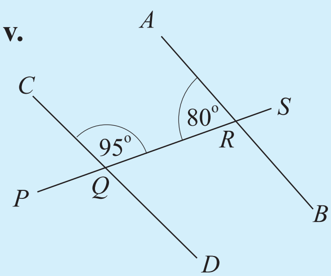
iii.



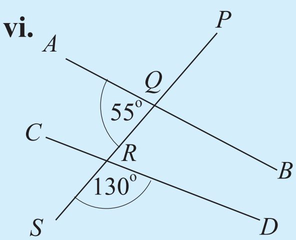
iv.



v.

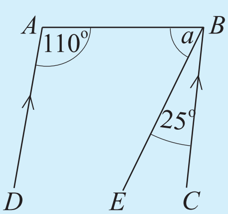


vi.

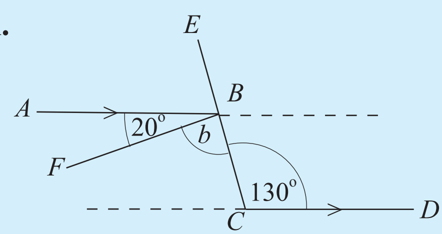


3. Find the value of each angle denoted by a lowercase English letter in the figures given below.

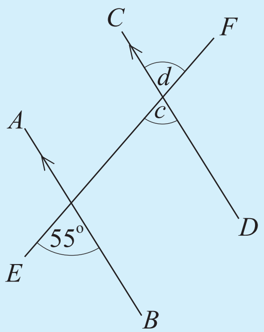
i.



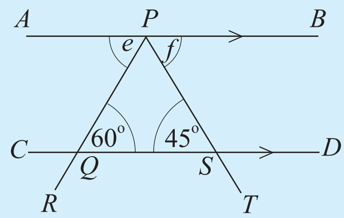
ii.



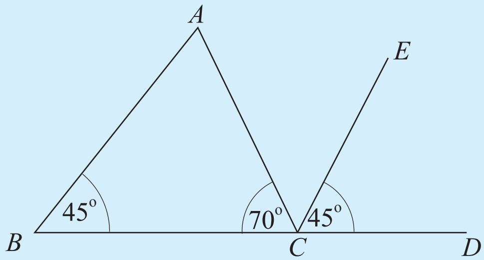
(iii)



(iv)



4.

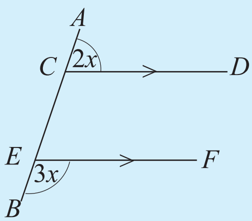


Based on the information given in the figure, show that $AB//EC$.

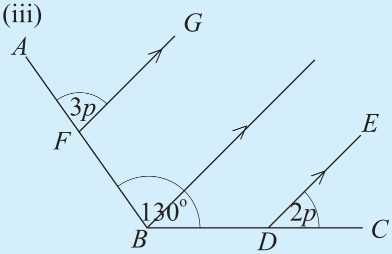
Miscellaneous Exercise

1. Find the magnitude of each of the angles denoted using lowercase English letters in the following figures.

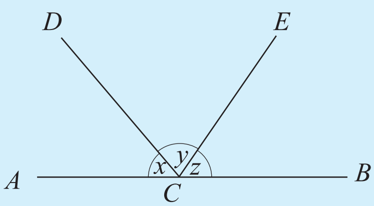
(i)



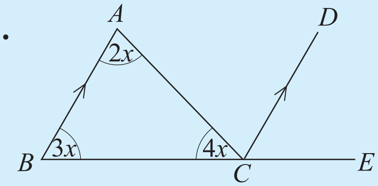
(iii)



2.



In the figure, x, y and z denote the magnitude of the relevant angle. If $x + z = y$, find the value of y.

3. 

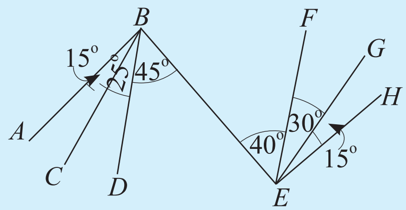
Based on the information in the figure,

(i) write the values of $D\hat{C}E$ and $A\hat{C}D$ in terms of x.

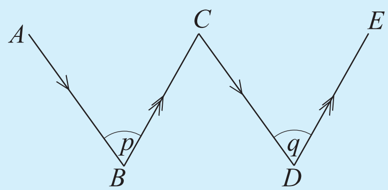
(ii) find the value of x,

(iii) find the magnitude of each angle in the triangle.

4. Write all the pairs of parallel lines in the given figure, indicating the reasons for your selections.



5. In the figure, $A\hat{B}C = p$ and $C\hat{D}E = q$. Show that $p = q$.



Summary

Summary

- The sum of the adjacent angles formed by a straight line meeting another straight line is two right angles.
- The vertically opposite angles formed by the intersection of two straight lines are equal.
- When two straight lines are intersected by a transversal, if
  i. a pair of corresponding angles are equal or
  ii. a pair of alternate angles are equal or
  iii. the sum of a pair of allied angles is 180°, then the two straight lines are parallel to each other.
- When a transversal intersects a pair of parallel lines,
  i. the corresponding angles formed are equal,
  ii. the alternate angles formed are equal,
  iii. the sum of each pair of allied angles formed equals two right angles.

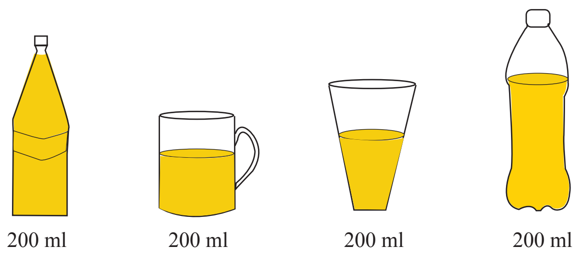
9 Liquid Measurements

By studying this lesson, you will be able to;

- determine the relationship between
  milllilitres (ml) and cubic centimetres (cm^3),
  litres (*l*) and cubic centimetres (cm^3),
  litres(*l*) and cubic metres (m^3), as units which are used to measure liquid volumes, and
- solve problems involving units which are used to measure liquid volumes.

Volume and capacity

We know that the amount of space occupied by a solid or a liquid is known as its volume.
A solid has a definite shape and a definite volume. Although a liquid has a definite volume, it does not have a definite shape. A liquid always takes the shape of its container.
The below given pictures show 200 ml of drink in different shaped containers



When that quantity of drink is poured into the different shaped containers, even though the liquid takes the shape of the container, its volume of 200 ml remains unchanged. In the first picture, the container is completely filled with the 200 ml of drink. Therefore, the capacity of that container is 200 ml. The **capacity** of a container is the maximum volume that it can hold.

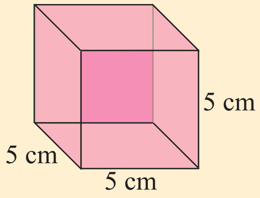
Do the following review exercise to recall the facts that you have learnt earlier in relation to volume and capacity.

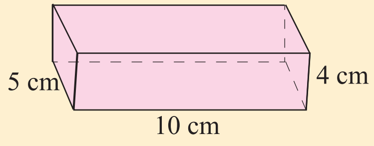
Review Exercise

1. Complete the table given below using the fact that 1 *l* =1000 ml.

ml	*l* and ml		*l* (in decimal form)
	l	ml	
2500	2	500	2.5
....	3	000	
3500	3		
....	4	500	4.5
....	0	500	
200			
50			
....			3.25
....	0	25	
....			0.005

2. Complete the two tables given below based on the way the volumes of the cube and the cuboid in the figure have been calculated.





Volume = 5 cm × 5 cm × 5 cm = 125 cm^3

Volume = 10 cm × 5 cm × 4 cm = 200 cm^3

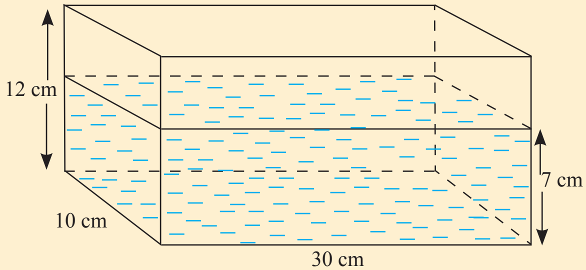
(i) Cube

The length of a side (cm)	Volume (cm^3)
2	.... × .... × .... = ...
4	
6	
7	
8	
10	
12	

(ii) Cuboid

Length (cm)	Width (cm)	Height (cm)	Volume (cm^3)
3	2	2	... × ... × ... = ...
5	3	4	
8	6	5	
10	5	10	
10	5	6	
12	10	8	
12	6	5	
15	8	10	
20	7	8	

3. The internal length, width and height of the container in the figure are 30 cm, 10 cm and 12 cm respectively. This container has been filled with water up to a height of 7 cm.



Determine the following.

i. The capacity of the container.

ii. The volume of water required to fill the whole container.

iii. The volume of water in the container, if the container is filled with water only up to a height of 7 cm.

iv. When the water level is 7 cm, if due to a leak it decreases to 5 cm within an hour, the volume of water that has leaked out during that hour.

9.1 The relationship between a cubic centimetre and a millilitre

A syringe used by doctors is given in the above figure. The amount of liquid medicine injected into a patient can be identified using the scale indicated on the syringe.

The units of measurement are indicated as cc/ ml.

cc means "**cubic centimetre**". It consists of the initial letters of these two words. A cubic centimetre is the volume of a cube of side length 1 cm.

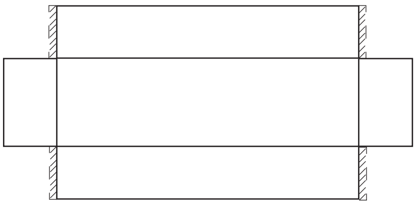
The back slash (/) means "or". It indicates that the amount of medicine can be expressed in terms of either cc or ml. The question which arises immediately is whether 1 cc is equal to 1 ml. In the metric system, 1 ml is defined such that it is equal to1 cc. Accordingly,

1 cubic centimetre = 1 millilitre

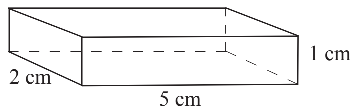
$$1 \text{ cm}^3 = 1 \text{ ml}$$

Do the following activity in order to study this fact further.

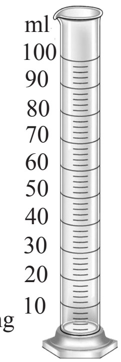
Activity



The net to construct the container



The cuboid shaped container of volume 10 cm^3



Measuring cylinder

- Construct a container of dimensions 5 cm × 2 cm × 1 cm using a net prepared from a thin plastic sheet as shown in the figure (paste the edges properly using sellotape or a suitable glue so that there is no water leakage).
- Obtain a measuring cylinder of capacity 100 ml from the laboratory.
- Draw the below given table in your exercise book.

Number of times water is poured from the cuboid shaped container into the measuring cylinder	The volume of water poured into the measuring cylinder	
	In cm^3 according to the cuboid shaped container	In ml according to the measuring cylinder
	10	
	20	
	30	
	40	
	50	

- Fill the cuboid shaped container completely with water and pour that water into the measuring cylinder.
- After pouring the water into the measuring cylinder, note down its reading.
- Repeat this process several times. Note down the reading on each occasion.
- Determine a relationship between the units of the volume of the container (cm^3) and the units (ml) marked on the measuring cylinder.

Based on the activity the following equalities are obtained.

$$10 \text{ cm}^3 = 10 \text{ ml}$$
$$20 \text{ cm}^3 = 20 \text{ ml}$$

Accordingly, it is clear that $1 \text{ cm}^3 = 1$ ml.

This relationship can be used when solving problems related to liquid volumes in containers.

Example 1

A cuboid shaped glass container of length 20 cm, width 15 cm and height 10 cm is filled with a liquid medicine.

i. Find the volume of the container in cubic centimetres.
ii. What is the capacity of the container in litres?
iii. If the liquid in the container is to be stored in phials of capacity 50 ml each, find the number of phials required to store all the liquid in the container.

i. The volume of the container $= 20 \text{ cm} \times 15 \text{ cm} \times 10 \text{ cm}$
$= 3000 \text{ cm}^3$

ii. The capacity of the container = 3000 ml
$= 3\ l$

iii. The total amount of liquid = 3000 ml

The number of phials of capacity 50 ml each that are required $= 3000 \div 50$
$= 60$

Example 2

800 l of water is in a cuboid shaped concrete tank consisting of a base of length 2 m and width 1 m. Find the height to which the water is filled in the tank.

Let us construct an equation, assuming that the tank is filled up to a height of x cm, and by solving it find the height to which it is filled.

Let us first convert all the measurements into centimetres.

The length of the tank = 2 m = 200 cm
The width of the tank = 1 m = 100 cm

The volume of water in the tank if the water level is x cm = 200 cm $\times$ 100 cm $\times$ x cm
$= 20\,000x \text{ cm}^3$

It is given that the volume of water in the tank is 800 l.

$\therefore$ Volume of water in the tank $= 800\ l$
$= 800\,000$ ml
$= 800\,000 \text{ cm}^3$

Since the volume of water represented above in two ways is equal,

$$20\,000 \times x = 800\,000$$

$$x = \frac{800\,000}{20\,000}$$
$$= 40$$

$\therefore$ The height of the water in the tank is 40 cm.

Exercise 9.1

1. Join each of the volumes in box A with the volume in box B which is equal to it.

A	B
1000 cm^3	25 ml
10 cm^3	25 l
3000 cm^3	1 l
1500 cm^3	10 ml
25000 cm^3	1.5 l
25 cm^3	3 l

2. The dimensions of several cuboid shaped containers are given in the following table. Complete this table.

Length (cm)	Width (cm)	Height (cm)	Capacity		
			cm^3	ml	l
20	10	5			
40	20	10			
35	12	10			
50	35	12			
40	35	25			
25	20	18			

3. A cuboid shaped tank of base area 240 cm^2 is filled up to a height of 12 cm with water. Find the volume of water in the tank in,

i. cubic centimetres **ii.** millilitres **iii.** litres

4. A cuboid shaped container has a square base of area 225 cm^2. An amount of 3.6 l of water has been filled into this container.

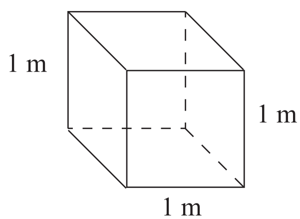
   i. Find the height of the water in the container.
   ii. If the height of the container is 24 cm, show that the water is filled to $\frac{2}{3}$ of its capacity.

5. Show that a barrel of capacity 15 l can be filled completely by pouring water 15 times using a completely filled cube shaped container of side length 10 cm.

9.2 The relationship between a litre and a cubic metre

The need for a unit which is larger than ml or l arises when it is necessary to measure large volumes of liquid such as the quantity of liquid in an oil tank or a swimming pool. In such instances a large unit called cubic metre is used.

In order to identify a cubic metre, let us calculate the capacity of a cube shaped container of side length 1 m.



The capacity of the container shown in the figure $= 1\text{ m} \times 1\text{ m} \times 1\text{ m} = 1\text{ m}^3$

However, since $1\text{ m} = 100\text{ cm}$,

the capacity of the container, $1\text{ m}^3 = 100\text{ cm} \times 100\text{ cm} \times 100\text{ cm}$

$= 1\,000\,000\text{ cm}^3$

$= 1\,000\,000$ ml (Since $1\text{ cm}^3 = 1$ ml)

$= \frac{1000\,000}{1000}\ l$ (Since 1000 ml $= 1\ l$)

$= 1\,000\ l$

Accordingly,

one cubic metre is equal to 1000 *l*.

$1 \text{ m}^3 = 1000\ l$

Example 3

The internal length, width and height of a cuboid shaped tank in which water is stored for the daily use of a household are 1.5 m, 1 m and 1 m respectively.

(i) What is the capacity of the tank in litres?
(ii) If the residents use 300 *l* per day, for how many days will a completely filled tank be sufficient?

(i) The capacity of the tank $= 1.5 \text{ m} \times 1 \text{ m} \times 1 \text{ m}$
$= 1.5 \text{ m}^3$
$= 1\,500\ l$ (Since $1 \text{ m}^3 = 1000\ l$)

(ii) The volume of water used per day $= 300\ l$
The volume of water in the tank $= 1\,500\ l$
$\therefore$ The number of days for which the water is sufficient $= \dfrac{1\,500}{300}$
= five days

Exercise 9.2

1. Complete the table.

The inner dimensions of the cuboid shaped tank			The capacity of the tank	
Length (m)	Width (m)	Height (m)	m^3	l
2	2	1	.....	.....
2	1.5	1	.....	.....
1	1	0.5	.....	.....
4	1	.....	8	.....
.....	1.5	3.0	.....	9000
1	.....	1	1.5	.....

2. The length, width and depth of a swimming pool are 50 m, 25 m and 3 m respectively.

   i. Find the capacity of the swimming pool

   ii. If the swimming pool is filled with water up to a height of 1.2 m, what is the volume of water in the swimming pool in litres?

   iii. How much more water is required to fill the swimming pool completely?

3. A bowser of capacity 6.5 m^3 is filled completely with oil. This bowser is supposed to distribute 850 l of oil each to 8 filling stations. Is the oil in the bowser sufficient for all 8 filling stations? Give reasons for your answer.

4. A person requires on average 150 l of water daily. If a cuboid shaped tank of length $1\frac{1}{2}$ m, width 1 m and height 1 m is completely filled with water, for how many people in total will this quantity of water be sufficient for a day?

5. The length of an interior side of a cube shaped tank is 1 m. The tank is completely filled with water. When a tap from which the water in the tank is discharged is opened, water flows out from the tank at a constant rate of 50 l per minute. Determine how long after the tap is opened the tank becomes empty, if the water flows out at this constant rate.

Miscellaneous Exercise

1. The capacity of a large drink bottle is 1.5 l. It is expected to serve a quantity of 150 ml of this drink in small glasses to each of the guests at a party. If there are 225 guests at the party, find the minimum number of large drink bottles needed to serve all the guests.

2. Household storage tanks of capacity 500 l, 1000 l and 2000 l are available for sale. The head of a family of 5 members intends to buy one of these tanks to store water for their household use. If each family member requires a maximum of 150 l per day and 200 l of water is required each day for other household chores. Determine which tank best suits his requirements, if the head of the family intends to fill the tank only once a day,

Summary

Summary

- 1 cm^3 = 1 ml
- 1 m^3 = 1000 l

Revision Exercise – First term.

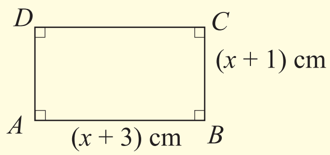
Part – I

1. Write the general term of the number pattern 5, 8, 11, 14, …, with a common difference.

2. Fill in the blank: 10011_{two} – .... $_{two}$ = 0011_{two} .

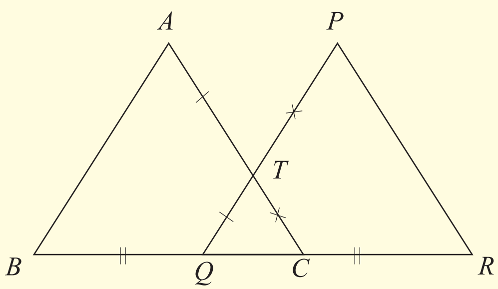
3. The value of $\frac{1}{3}$ of a certain amount of money is Rs 800. What is the value of $\frac{3}{4}$ of that amount of money?

4. If a profit of Rs 300 is earned by selling an item for Rs 1500, what is the profit percentage?

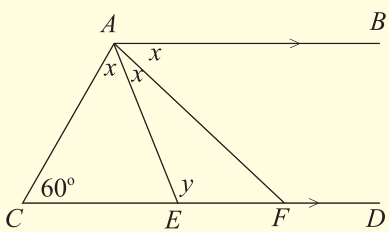
5. 

Express the area of the rectangle $ABCD$ in terms of x.

6. Factorize $2x^2 - x - 6$

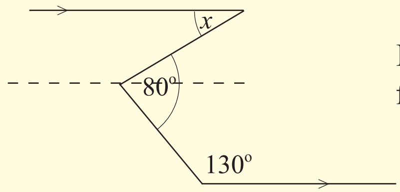
7. 

Show using axioms and the information in the figure that,
(i) $AC = PQ$ and
(ii) $BC = QR$.

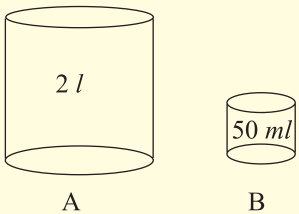
8. 

Given that the lines AB and CD are parallel, find the value of y.

9. Find the values of b and c if $(x + 4)(x - 3) = x^2 + bx + c$.

10. 

Find the value of x using the information in the figure.

11. 

How many times should water be poured into the container A of capacity $2l$ from the completely filled container B of capacity 50 ml to fill $\frac{3}{4}$.

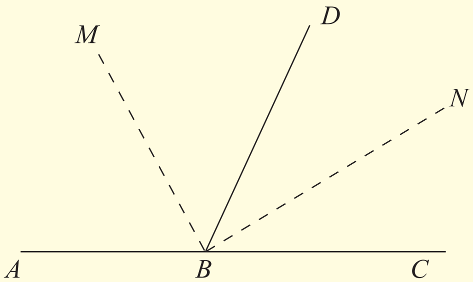
12. brokerage fee of 3% is charged when a land is sold. If the land owner received 27 lakhs in rupees after the brokerage fee was paid, find the price at which the land was sold.

13. What is the fraction by which $1\frac{3}{4}$ has to be multiplied to obtain $3\frac{3}{4}$?

14.

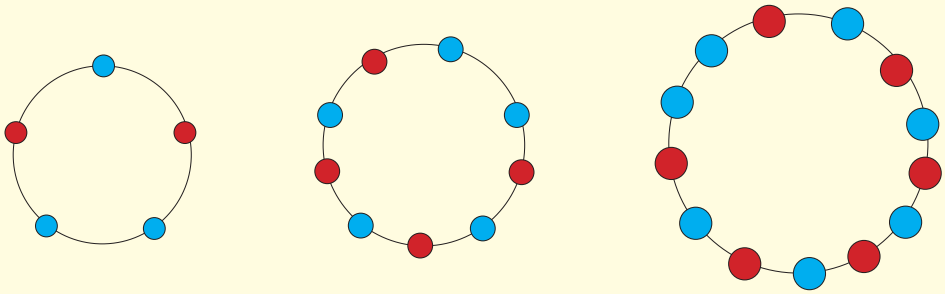
$$\begin{array}{r} 1101_{two} \\ +\ 1111_{two} \\ \hline \ldots\ldots\ldots\ldots \\ -\ \ 101_{two} \\ \hline \ldots\ldots\ldots\ldots \\ \hline\hline \end{array}$$

Fill in the blanks.

15. 

The bisectors of $A\hat{B}D$ and $D\hat{B}C$ are BM and BN respectively. Find the value $A\hat{B}M + C\hat{B}N$, if $A\hat{B}C$ is a straight line.

Part II

1. 

1. Adecoration is madeby preparing circles of various sizes and placing red and blue bulbs in a pattern with a common difference such that the first three arrangements contain 3, 5, 7 blue bulbs and 2, 4, 6 red bulbs respectively as shown in the figure.

   (i) Write the number of blue bulbs and the number of red bulbs in the 4th and 5th arrangements.

   (ii) Identify the patterns of the number of blue bulbs and the number of red bulbs in the arrangements and construct two expressions in n for the number of bulbs of each colour in the nth arrangement.

   (iii) Find an expression for the total number of bulbs inthe nth arrangement, using the expressions in (ii) above.

   (iv) Find the number of blue bulbs and the number of red bulbs in the 10th arrangement using the expressions in (ii) above.

   (v) Which arrangement is prepared using a total of 61 bulbs? Find the number of blue bulbs in that arrangement.

2. (a) Simplify.

i. $$\frac{2\frac{1}{5} + \frac{1}{2}}{\frac{3}{10}}$$

ii. $(1\frac{1}{8} \text{ of } 1\frac{1}{3}) \div 2\frac{1}{2}$

(b) **i.** $\frac{1}{4}$ th of a certain land contains mango trees. What fraction of the total land is the remaining portion of land?

ii. If $\frac{1}{3}$ of the remaining land contains banana trees, express the portion of land in which banana trees are grown as a fraction of the whole land.

iii. In what fraction of the total land are the mango trees and banana trees grown?

iv. If the area of the portion in which these trees are not grown is 3 hectares, what is the total area of the land?

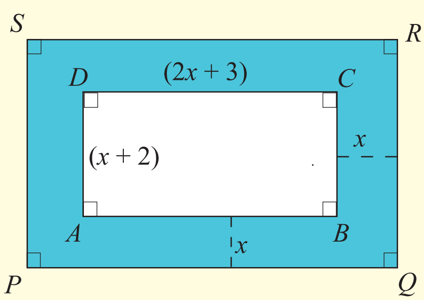
3. (a) The selling price of an item which was bought for Rs 8000 was marked keeping aprofit of 25%. A discount of 10% was given when the item was purchased outright. Find the profit percentage earned by the seller.

(b) A person marks the price of an item to earn a profit of 15%. If its price had been marked to earn a profit of 20%, an extra Rs 200 could have been earned. Find the purchase price and the marked price of the item.

4. (a) Find the value of each of the following expressions when $a = -2$ and $b = 3$.

i. $2a + 3b$ **ii.** $b - 2a$ **iii.** $\frac{a}{3} - \frac{b}{2}$

(b) a



The rectangle $ABCD$ in the figure represents picture. Its length is $(2x+3)$ cm and its breadth is $(x + 2)$ cm.

i. Obtain an expression in terms of x for the area of $ABCD$.

ii. he shaded part in the figure represents a band of breadth x cm which is pasted bordering $ABCD$. Find an expression for the area of the rectangle $PQRS$ and express the area of the shaded part in terms of x using the expression found in (i) above too.

iii. Calculate the area of the shaded part if $x = 3$ cm.

(c) Factorize the following expressions.

i. $5x^2 + 12y^2 - 4xy - 15xy$

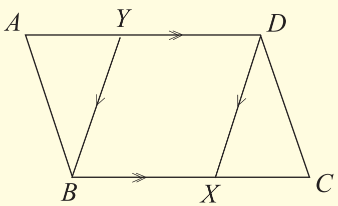
ii. $6\,(x - 1) + 3x - 3$

iii. $t^2 - 8t + 15$

iv. $3k^2 - 12k$

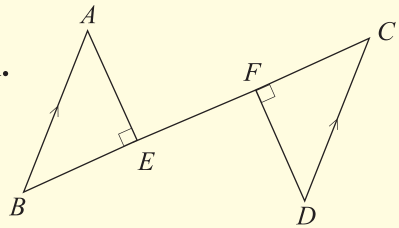
5. (a) Obtain the following results using axioms and the information in the figure.

i.



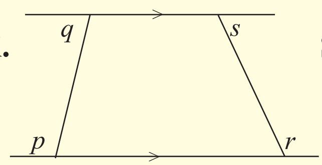
Show that $A\hat{Y}B = D\hat{X}C$.

ii.



Show that $B\hat{A}E = F\hat{D}C$.

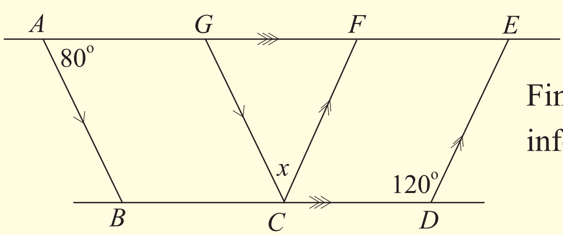
iii.



Show that $\hat{p} - \hat{s} = \hat{r} - \hat{q}$.

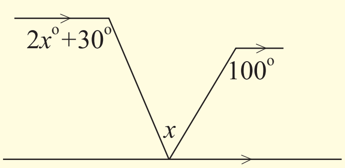
(b)

i.



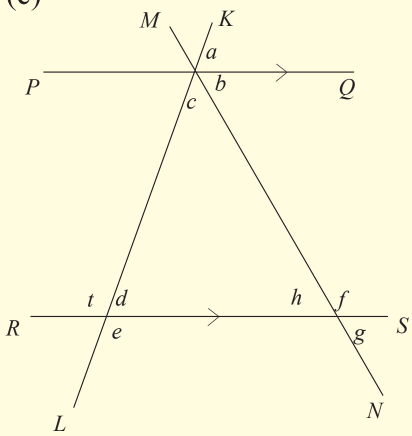
Find the value of x using the information in the figure.

ii.



Find the value of x using the information in the figure.

(c)



The parallel lines PQ and RS are intersected by the transversals MN *and* KL. Answer the following questions using the information in the figure.

i. Write all the instances where the sum of the given angles is 180^{o}.

ii. Write the pairs of allied angles in the figure.

iii. Providing reasons, write all the angles which are equal to each other.

iv. Is $\hat{a} + \hat{e} = 180^{o}$? Explain your answer.

v. Using axioms, show that $t - f = h - d$.

vi. Find the values of all the angles indicated by lowercase letters if $e = 140^{o}$, $f = 110^{o}$.

6. The length, breadth and height of a water tank are 2m, 1.5m and 1m respectively.

i. Express the capacity of the tank in liters.

ii. If the daily water requirement of a person is 150 *l*, how much water is required daily for 4 people?

iii. For how many days will the water in this tank be sufficient for4 people if it is full?

iv. If water is supplied to the tank at a rate of 100 *l* per minute, how much time is needed to fill thetank if it is empty?

v. On a day when the tank is filled to its capacity, 900 *l* of water leaks out due to a fault in the delivery pipeline. Find the height of the remaining water.

Glossary

A

Addition	එකතු කිරීම	கூட்டல்
Allied angles	මිත්‍ර කෝණ	நேயக்கோணங்கள்
Algebraic expressions	වීජීය ප්‍රකාශන	அட்சரகணிதக் கோவைகள்
Algebraic term	වීජීය පදය	அட்சரகணித உறுப்பு
Alternate angles	ඒකාන්තර කෝණ	ஒன்றுவிட்டகோணங்கள்

B

Base	පාදය	அடி
Binary numbers	ද්වීමය සංඛ්‍යා	துவித எண்கள்
Brackets	වරහන්	அடைப்பு
Broker	තැරැව්කරුවා	தரகர்

C

Capacity	ධාරිතාව	கொள்ளளவு
Commission	කොමිස්	தரகு (கமிஷன்)
Common factors	පොදු සාධක	பொதுக்காரணிகள்
Converse	විලෝමය	மறுதலை
Conversion	පරිවර්තනය	மாற்றல்
Corresponding angles	අනුරූප කෝණ	ஒத்தகோணங்கள்

D

Difference of terms	පද අතර වෙනස	உறுப்புக்களுக்கிடையேயானவித்தியாசம்
Discount	වට්ටම	கழிவு

F

First term	පළමුවන පදය	முதலாம் உறுப்பு
Fractions	භාග	பினன் ஙக் ள்

G

General term	සාධාරණ පදය	பொது உறுப்பு

I

integers	නිඛිල	நிறைவெண்கள்

L

Loss	අලාභය	நட்டம்

M

Marked Price	ලකුණු කළ මිල	குறித்த விலை

N

Number sequence	සංඛ්‍යා අනුක්‍රම	எண் தொடரி

P

Place Value	ස්ථානීය අගය	இடப்பெறுமானம்
Power	බලය	வலு
Profit	ලාභය	இலாபம்

S

Sciencetific notation	විද්‍යාත්මක අංකනය	விஞ்ஞானமுறைக் குறிப்பீடு
Selling Price	විකුණුම් මිල	விற்றவிலை
Subtraction	අඩු කිරීම	கழித்தல்

T

Theorem	ප්‍රමේයය	தேற்றம்

V

Vertically opposite angles	ප්‍රතිමුඛ කෝණ	குத்தெதிர்க்கோணங்கள்
Volume	පරිමාව	கனவளவு

Lesson Sequence

Contents	Numbers of Periods
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